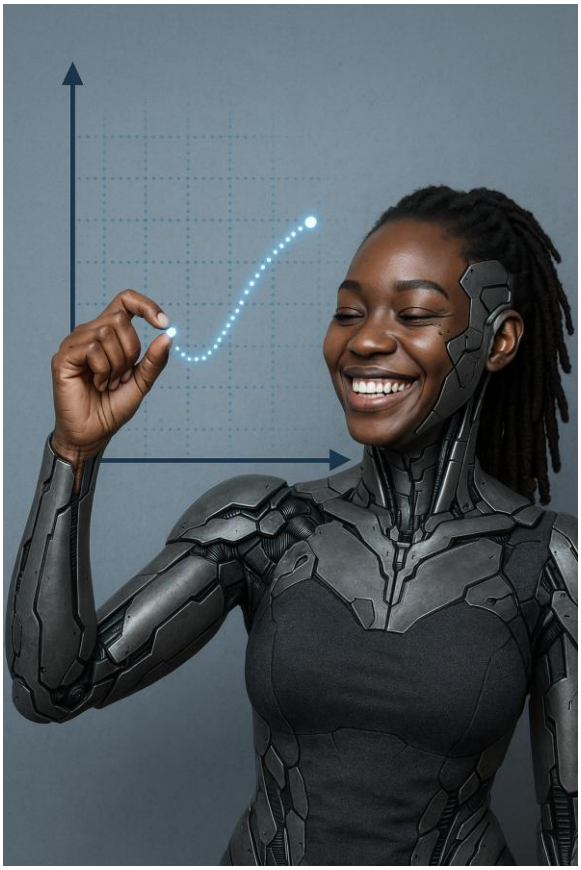
Introduction to Impossible Probability,

Statistics of Probability or

Probabilistic Statistics. VOL 2

Dr Rubén García Pedraza



Introduction to Impossible Probability,

Statistics of Probability or Probabilistic Statistics

Dr Rubén García Pedraza

2015, Revised Edition, for Print and eBook Version.

Translated into English in September 2025,

using ChatGPT, keeping the translation as faithful as possible to the original

Preliminary Words [2015 edition]

Introduction to Impossible Probability, Statistics of Probability or Probabilistic Statistics is one of those works in which a deep bond between mathematics and philosophy can be found.

It always asks about the ultimate purpose of science, wrapped in a veil of uncertainty and relativity.

There is always a halo of indeterminism, since in the end it is chance itself—the random variations of what we call reality, although we do not quite know what it is—that becomes the cause of causes.

From an interpretation not devoid of a certain nihilism, it offers a different vision of mathematics applied to statistics and probability.

The work is organized into a series of sections whose exposition unfolds in a continuous manner, each section and each paragraph being numbered in an ordered numerical sequence, in a way that facilitates reading.

Introduction to Impossible Probability first appeared in December 2011, in a limited edition of which a few copies still remain. It was later modified into a new eBook version that appeared in January 2013, whose first revision dates from January 2014. The version now presented is the latest update, based on contributions from January 2015.

To all readers, many thanks, and enjoy,
Rubén García Pedraza

Preliminary Words to the English Translation

The work presented here, *Introduction to Impossible Probability*, is essential for understanding the original meaning of the Global Artificial Intelligence. Its origins can be traced back to my writings in 2002 on Astro-ecology, the science that understands the cosmos as a single environment that must be studied from a unified, single science. In essence, this is what the Global Artificial Intelligence seeks to achieve. At the same time, in 2002 I developed the formulations of Impossible Probability, beginning in the early hours of September 11, 2001 with the *Impact of the Defect*, which was later expanded into the *Second Method* on October 16, 2002.

In essence, the entire original philosophy is based on the Cartesian idea of the necessity of a margin of doubt in any scientific proposition. This Cartesian margin of doubt would later, in the application of Karl Popper's falsificationism to inferential statistics, be understood as *critical reason*, the necessary and acceptable margin of error in any provisionally true proposition.

Impossible Probability maintains and deepens the Cartesian and Popperian rationalist tradition, enriching it with new philosophical contributions and some mathematical formulations. It rests on the understanding that the necessity of a margin of doubt in the existence of every provisional truth stems from the human limitation of being incapable of accessing the noumenon of things. It is this a priori limitation—the metaphysical limitation of the human condition—that requires humans to accept errors. Yet this does not mean that the noumenon cannot be known by other means. For this reason, in *Impossible Probability* we propose the construction of the Global Artificial Intelligence: a machine capable of making real the dream of a unified science, a single science that may lead us to the reason for all things. The creation of a global matrix, which may ultimately be resolved in a single equation, will provide the equation that solves the matrix, the very reason for everything.

Due to the special conditions in which I find myself—explained in other documents available on Internet Archive and Amazon—I am compelled to publish as quickly as I translate. For this reason, the English version of *Introduction to Impossible Probability* will be released in volumes as the translation progresses.

September 1, 2025, London, Leytonstone, 18:32

Imposiblenever@gmail.com

“What did you do with the other guys

Did people brush their teeth

In the concentration camp”

Ruben Garcia Pedraza

Poem “*Mr Saddle”*

At Stratford Library Poetry Club

Table of contents Vol 2.

4. First Method

5. The Second Method of Impossible Probability

6. Validity and Sample Level

4.First Method

4.1. The analytical method of Impossible Probability for developing the propositions of the statistics of probability and probabilistic statistics is the syllogism of the trend. From this analytical method, the different methods of the statistics of probability or probabilistic statistics are developed to describe or critique empirical reality. These methods that Impossible Probability develops to describe or critique empirical reality are the Impact of the Defect, the Second Method, the Effective Distribution, as well as other methods explained in this work, adapting the rational models for critiquing empirical reality to models of raw scores or frequencies, or to ranking models.

4.2. Thus, we must distinguish between the analytical method of Impossible Probability—the syllogism of the trend—and the synthetic methods, namely those by which the statistics of probability or probabilistic statistics carry out a synthetic function of a descriptive or inferential character, and within the inferential, for the rational critique of empirical hypotheses, whether explanatory empirical hypotheses or technological empirical hypotheses, both in the social and natural sciences.

4.3. The analytical method, the syllogism of the trend, is deductive in nature, and the synthetic method is the hypothetico-deductive method; therefore, both the analytical and the synthetic methods must alike be deductive. The only difference is that the analytical method is a deductive-analytical method, and the synthetic method is a deductive-synthetic method; consequently, synthetic-deductive hypotheses must be subjected to critical reason in order to be accepted as provisionally universal.

4.4. The reason why the synthetic method called the Second Method receives this name is that it is an alternative method—a Second Method—for studying the empirical data

that samples can provide us, via empirical probability and theoretical probability, different from the way traditional statistics (the First Method) does it. In traditional statistics, the mean, mode, median, mean deviation, variance, standard deviation, and standard score are always obtained directly from raw scores or frequencies. In the Second Method, by contrast, all—and absolutely all—statistics, for any kind of universe, infinite or limited, are obtained from empirical probabilities.

4.5. Before moving on to explain the Second Method, a brief aside to describe the most characteristic features of the First Method—for example, the arithmetic mean. Whenever in Introduction to Impossible Probability the summation symbol "∑" appears before a bracket or parenthesis, it denotes summing what is inside the bracket or parenthesis; and if, after that summation, there appears a division by some factor, then the sum of the bracket or parenthesis is divided by that factor. In this way, the traditional equation for the arithmetic mean in the First Method is as follows: the sum of the product of each raw score and its frequency, divided by the total frequency

$$\text{Arithmetic Mean (First Method)} = \sum(f_i \cdot x_i) / \sum f_i$$

4.6. For Impossible Probability, such explanations of the arithmetic mean are in any case descriptive. The significance of Impossible Probability is that probability, as an estimate of the possible future, has predictive value in and of itself: it does not only describe, it allows value judgments about possible future events. The traditional definition of the arithmetic mean has only a descriptive function, whereas the inverse of N also fulfills a predictive function in the Second Method. The difference between conventional mathematics and the Second Method of Impossible Probability lies in this: Impossible Probability is a statistics of probability—whether in the Second Method of October 16, 2002, in the Impact of the Defect of September 11, 2001, or in the Effective Distribution of the summer of 2003. In Impossible Probability, every description based on probabilities has a predictive function in itself. In Impossible Probability, the arithmetic mean of the empirical probabilities, in any kind of universe, is the inverse of N, 1/N, serving the predictive function of what the random behavior of

any subject or option in the sample should be, whether in universes of infinite subjects or options, or in universes of limited options, under equality of opportunity.

4.7. If the arithmetic mean is the sum of the products of scores by frequencies, divided by the total frequency, then the mean deviation equals the sum (divided by the total frequency) of the absolute values of the differential scores. The sum of the absolute values of the differential scores equals the sum of the absolute value of each differential times its frequency. Whenever a differential is placed between two bars like "|(A − B)|" it means the result is taken without sign, i.e., the absolute value of the differential.

$$MD\ (\text{First Method}) = \sum[\ fi \cdot |xi - \bar{x}|\ ] / \sum fi$$

MD = Mean Deviation

4.8. Obviously, the variance is the same but with the differential squared:

$$S^2\ (\text{First Method}) = \sum[\ fi \cdot (xi - \bar{x})^2\ ] / \sum fi$$

S^2 = Variance

4.9. Logically, the standard deviation is the square root of the variance:

$$S\ (\text{First Method}) = \sqrt{\{\ \sum[\ fi \cdot (xi - \bar{x})^2\ ] / \sum fi\ \}}$$

S = Standard Deviation

4.10. Unlike traditional statistics, Impossible Probability understands mathematics dialectically. If the inverse of N, 1/N, or the inverse of the raw scores or frequencies, 1/∑xi, can be said to be important contributions of Impossible Probability to the study of what happens—reality—as a series of events, simultaneous, linear and non-linear, in a continuous curved space-time, within a complex multi-causal and multi-variable system that can be studied through the statistics of probability or probabilistic statistics, and statistics in general—chance and history—then all these innovations that Impossible Probability makes are essentially logical consequences of the logical application of dialectics to mathematics. If dialectical logic is established on the law of the dialectical identity of opposites, it has its origin in the transformation of the qualitative into the quantitative and vice versa—in other words, the transformation of reality into idea and vice versa, that is, the transformation of subject into object and vice versa. If, in statistics, for example, when we exercise our right to vote in an election in which a set of candidates or delegates is presented, or if we study a group of students to whom an intelligence-improvement program has been applied, we understand that the act of voting for one candidate or another, or the fact that each student in the sample is an object of the sample, implies that each candidate or delegate in the election can be interpreted as a subjective option, and the fact that a sample is formed by a set of students implies that each student is an optional subject of the sample, among the infinite optional students who could be included by chance in the sample—and we understand that every subject becomes an option and every option becomes a subject, the dialectic of the identity between subject and object—then, if we understand this dialectical identity, we can begin to understand Impossible Probability, the dialectic. Impossible Probability is simply the logical-dialectical application to mathematics. The analytical method of Impossible Probability is the syllogism of the trend.

5.Second Method of Impossible Probability

5.1. Introduction to the Second Method of Impossible Probability

5.1.1. The Second Method was created on October 16, 2002, within the essay *Intuition and Probability*, which I began on October 14, 2002 and finished at the end of October.

That essay had a second part during the winter of 2003, among whose main formulas the Level of Similarity stands out, to which I will refer later.

5.1.2. Every quotient is a rational number.

5.1.3. Probability is a quotient; therefore probability is a rational number.

5.1.3.1. Every quotient is the relationship between two factors.
5.1.3.2. The first factor is that which is divided (the dividend).
5.1.3.3. The second factor is that which divides (the divisor).
5.1.3.4. The first factor is that which is distributed, or that whose proportions one wishes to know. To measure is to give proportion to reality; a proportioned reality is a measured reality; everything proportioned is measured; to measure is to define the mathematical limits of reality.
5.1.3.5. The second factor is that in terms of which the unit of proportion is established; the unit of measurement is the second factor of the quotient. It is that in terms of which the limits of reality are established—the standard of measurement, the criterion of reality's limits.

5.1.4. Probability and proportionality

5.1.4.1. Every proportion is the proportion that several elements bear among themselves relative to a unit of measurement, in which each element to be measured is divided by the unit of measurement. When the length or width of a two-dimensional space is measured, one establishes the proportionality of how many units of measurement there are in that length or that width.
5.1.4.2. Every measurement is a proportion in which the second rational factor of the quotient is the unit of measurement.
5.1.4.3. Every measurement is a rationalization of reality in terms of a unit.
5.1.4.4. A unit [Parmenides, Plato?] may be a whole or simply a part, but a part insofar as it functions as a whole: if I say A is X meters, the unit of measurement is the meter; if I say A is X millimeters, the unit is the millimeter; if I say A lasts X hours, the unit is the hour; if I say A lasts X seconds, the unit is the second.
5.1.4.5. The first factor can likewise be a part or a whole, i.e., a unit—the unit of a part of the whole or the unit of the whole—where the kind of measurement unit for the first

factor need not correspond to that of the second factor. For example, the unit for the first factor can be kilometers and for the second factor hours, with the result being kilometers per hour.

5.1.4.6. If the number in the first factor equals the number in the second factor, the quotient equals one (the unit).

5.1.4.7. The quotient of a part of the whole divided by the whole equals a proportion. A probability is the decimal value of the proportion between the part and the whole. Empirical probability equals the raw score or frequency divided by the sum of raw scores or frequencies, yielding the proportionality of the part to the whole.

5.1.4.8. The quotient of the unit and the whole equals the inverse of the whole, and it measures the proportion of each part of the whole in the total unit. An example of this quotient in the Second Method is the inverse of N, 1/N, or the inverse of the raw scores or frequencies, $1/\sum x_i$.

5.1.4.9. The quotient of one part by another part is the proportion between parts. One example, in a growth study, is dividing the empirical probability of a second measurement of a given subject or option by the empirical probability of the first measurement of that same subject or option; or, in a decay study, likewise dividing the empirical probability of the second measurement by that of the first. The quotient of two empirical probabilities, in two different measurements of the same subject or option, will allow one to know the slope in the rate of growth or decay of that subject or option—that is, a simple derivative.

5.1.5. Every proportional probability of a part over the whole, multiplied by 100, is a percentage, and mathematically it is used to describe proportions.

5.1.6. Since any probability multiplied by 100 is a percentage, every probability is a proportion between the part and the whole.

5.1.7. Every probability, as a probability, has predictive value about possible future behavior; as a percentage it has descriptive value about reality; and, being both descriptive and predictive, it can be used to understand and explain reality and to make decisions about reality in line with future predictions of possible occurrence.

5.2. The sample

5.2.1. In Impossible Probability there are two sample models: the sample of N subjects or options, and the sample of raw scores or frequencies.

5.2.1.1. The N sample

5.2.1.1.1. In a universe of infinite subjects or options, the sampling of subjects or options is the sample of subjects or options, made up of N subjects or options, with N being the total number of subjects or options. In universes of limited options, the options sample is limited to the N options, the N options being the sample of options. Each subject or option in the N sample, in any kind of universe—of infinite subjects or options, or of limited options—will have an associated raw score or frequency; the quotient of its raw score or frequency over the sum of raw scores or frequencies will be the empirical probability of the subject or option. The Second Method will use the concept of empirical probability of a subject or option interchangeably in either universe—of infinite subjects/options or of limited options; for any kind of universe the empirical probability of a subject or option is the same.

5.2.1.1.1.1. N options, in samples from universes of limited options. If we are in a limited-options model, then the number of options will be the number N, where N equals the number of options. The N sample will be limited to the number of options; N will be the sample of possible options.

5.2.1.1.1.1.1. If we toss a coin, normally there are only two options—heads or tails—so N = 2, i.e., a sample of two options, a universe limited to only two options, heads or tails.

5.2.1.1.1.1.2. If we play "which number will come up" when throwing a single dice, then, assuming a dice has six faces and there are only six options, N = 6, since the options sample equals six options—the number of options in the sample—i.e., a universe of options limited to the dice's six options.

5.2.1.1.1.1.3. If, in a democratic election, a number of candidates stand, N will equal the number of candidates running; the universe of possible candidates is limited to those who stand for election.

5.2.1.1.1.2. N subjects or options. If we are in a model of N subjects or options, the subjects will be treated as if they were likewise options; then the number of subjects or options will be the number N of subjects or options. The N sample of subjects or options will be limited by the number of subjects or options drawn from the universe of infinite subjects or options to form the sample. (Recall that universes of infinite subjects or options include complete population studies, where the population itself is the sample, with no sampling.)

5.2.1.1.1.2.1. If we are conducting research on the effectiveness of a vaccine in generating immune defenses by studying the empirical probability of symptoms in the sample after administering the vaccine, the sample of subjects or options will be the number of people—subjects or options—used in the various trials. For example, we will take one sample of people for the experimental group, to whom the vaccine is applied, and another sample of people for the control group, who will be injected with a saline solution. The N sample of subjects or options (people) in the experimental group should be the same size N as the control group. In this way, depending on the number of subjects or options—that is, on N—the sample will be more or less representative: the larger the number of people (subjects or options, N) in the sample, the more representative the sample; the smaller the number (N), the less representative it is. The larger N is in the experimental sample, the more representative it becomes, since the sample will include a greater number of different individuals, with different immune systems reacting to the vaccine in question; with more varied individuals and immune systems, the production of defenses in each person will differ, and the response to the vaccine can be studied more broadly. In the control sample it is also important to have a substantial N of people (subjects or options), at least equal to that of the experimental group, in order to know whether the reactions in the control sample, in the absence of the vaccine, among N people (subjects or options) with different immune systems and no treatment, evolve differently or not compared to the N people in the experimental sample.

5.2.1.1.1.2.2. In the previous example, to explain the concept of N subjects or options for which the raw score is to be measured on a particular quality, I used N subjects or options in the sense of people. But mathematically, the concept of N subjects or

options can be generalized to any possible sample for which it is necessary to measure the raw-score behavior of a particular quality in each singular member of that sample, drawn from the possible infinite universe of subjects or options, where one can choose N subjects or options to study the intensity of some measurable aspect, from a possible universe or population of subjects or options. If we are researching the behavior of a particular quality—measured as a raw score—in animals, then N equals the sample of animals used in the research; thus the N subjects or options will be the N animals used as subjects. If we study the behavior of stars, then the N stars will be the N sample subjects or options in the research—the N stars are the N subjects or options. If we conduct a study in a hadron collider, then the N collisions of subatomic particles we use can be the sample number N of subjects or options. It could also be that, instead of considering the N possible collisions, we use other parameters, such as the N particles involved in a collision, or any other parameter. What matters, in any case, in Impossible Probability is to define the N sample, because defining the N sample implies defining the object of study; and in the case of universes of subjects or options, the N sample is defined over the sample of subjects or options for which the intensity of a quality is measured as a raw score on an instrument. If we study a new medicine and define the object of study as the raw score of symptoms per subject or option, it will be a universe of infinite subjects or options; otherwise, if the universe is defined over the N possible outcome options—e.g., the result is positive or negative—then it would be a universe limited to two options: positive or negative. This implies that the type of universe is defined by the object of study, i.e., by whether what is being studied are frequencies over a set of limited options (a universe of limited options) or raw scores over a possible universe of infinite subjects or options.

5.2.1.1.1.2.3. The N sample in universes of infinite subjects or options: a subject or option, in such universes, is any subject liable to be optional among the possible optional subjects in an infinite universe, to study a particular quality of the universe through the raw scores of the subjects in the selected sample, the raw score being the individual measurement of how that particular quality of the universe manifests in that subject. The sample of subjects or options will be the N sample, and the number of subjects or options in the sample constitutes the number N in universes of infinite

subjects or options. Typically, in inferential statistics one tends to select an N sample; and in descriptive population studies one tends to include the entire population—for example in demographic studies—N being the total population. In any case, the larger N is—and the more the entire population is included—the smaller the sampling representativeness error. As the N sample shrinks, the sampling representativeness error tends to increase. As the population of Amazonian trees with medicinal uses tends to disappear, being endangered, the smaller the population N, the greater the sampling representativeness error for the population of Amazonian trees with medicinal uses. Population studies will thus be included among universes of infinite subjects or options. Only an appropriate international policy could make it possible for the population of Amazonian trees with medicinal uses to enjoy a potentially infinite history. Any possible population is one population among infinitely many possible populations across infinitely many instants in history. When studying one complete population among the infinitely many complete populations that could exist across infinite instants, the population, in the infinite, becomes a sample. In the infinite, everything is transformed because nothing is as it seems.

5.2.1.1.1.2.4. Characteristics of the N sample of subjects or options in universes of infinite subjects or options

5.2.1.1.1.2.4.1. The selection of sample subjects or options in universes of infinite subjects or options—the formation of the sample of subjects or options.

5.2.1.1.1.2.4.1.1. In a universe with a quantity of subjects or options tending to infinity—whether across infinitely many instants within a period during which measurements are made on the population, or in a potentially infinite history—Probability would be Impossible as infinity tends to zero. To resolve these problems, the researcher selects a sample; the selection should be random. The theoretically ideal sample is the random sample—the sample where the empirical behavior of the sample is a product of the behavior of chance within the sample. Intelligence is the capacity to solve problems; the problem of the infinite tending to zero and of the need for space-time (historical) limits to objectivity is solved by random selection of an N that is sufficient or rational—that is, the size N must be a sufficiently rational, or rationally sufficient, size, representative enough to accept the minimal morally necessary error.

5.2.1.1.1.2.4.1.2. Random selection and bias error

5.2.1.1.1.2.4.1.2.1. Definition of bias: in mathematics, bias refers to any tendency not explainable by chance; normally, any behavior different from what chance would yield is bias—biased behavior. Dialectically, as will be seen as the theory advances, bias and chance are identical because they are opposites. The Mean Deviation is the empirical distribution of Total Bias due to chance, the empirical product of the behavior of chance. The Mean Deviation is that part of Total Bias (the sum of the absolute values of the Bias Levels) explained by chance; the empirical distribution of Total Bias by chance equals Total Bias times the inversion of N—in short, bias by chance or chance by bias, the bias of chance or the chance of bias. The order of the factors does not change the product. Bias and chance are identical within the margin of error one is willing to accept— theoretically, the inversion of the sample; empirically, the empirical oscillation error around the central tendency.

5.2.1.1.1.2.4.1.2.2. A subject or option is said to have bias when its behavior is not equal to the theoretical probability of occurrence by chance, under equal opportunity—i.e., the inverse of N. The inverse of N is, in theory, the probability of occurrence by chance; theoretical probability is the ideal of equal opportunities for random occurrence. The subject's bias is what identifies the subject's individual behavior. The bias of each subject or option determines the individual identity of each subject or option. When a subject or option has an empirical probability different from the inverse of N, we say there is bias. In games of chance—dice, coin tosses, or roulette—bias indicates that the coin, dice, or wheel has been rigged to raise the probability of one option over the others. When, in an election, one candidate receives more votes than another, then the more-voted candidate has a positive bias among voters, and the less-voted candidate has a negative bias among voters. The positive bias of the more-voted candidate equals that candidate's positive normal Bias Level, and the negative bias of the less-voted candidate equals that candidate's negative normal Bias Level. The normal Bias Level is the differential score of empirical probability minus theoretical probability of random occurrence (the inverse of N). *Statistics is the study of history's behavior under chance*. History is the random sequence of accidents and contingencies—the logical sequence of events within the chance of history. History

crystallizes in accidents that determine the very contingency of real reality; the apparent determination of history is the way history appears. Based on the sample of what happens, one describes or infers, accepting a margin of error—empirical skepticism or rational doubt—within which causal indeterminism is possible.

5.2.1.1.1.2.4.1.2.3. The normal Bias Level of a subject or option is equal to the difference between the empirical probability and the theoretical probability of random occurrence. The normal Bias Level is the individual chance-bias. If the empirical probability is lower than the theoretical probability, the normal Bias Level is necessarily negative; and if the empirical probability is higher than the theoretical probability, the normal Bias Level is necessarily positive. The sum of all positive normal Bias Levels equals the sum of all negative normal Bias Levels, which is why Total Bias is the sum of all normal Bias Levels taken without sign—i.e., using only the absolute value of the normal Bias Level, regardless of whether it is positive or negative. The Mean Deviation is the distribution of Total Bias by chance, that is, Total Bias times the inverse of N—Average Bias Level. If the normal Bias Level is the individual chance-bias, the Mean Deviation is the sample chance-bias. Probability is the study of what happens; statistics is the history of chance; the Second Method studies what happens in chance and in history, whether natural or social [and we must include technological, the technological section of each hemisphere, factual and conceptual in the particular matrix, fifth phase, or the matrix in the sixth phase]. Impossible Probability is identical to positivism: all natural and social history must be investigated according to the hypothetico-deductive method— with the difference that in Impossible Probability every decision is a political decision. Impossible Probability is identical to positivism by being its opposite: for Impossible Probability, as a positivist theory, everything social is natural and everything natural is social [in the same way that in the end, everything is artificial, artificial is everything]. For Impossible Probability there is no difference between the positive natural sciences and the phenomenological social sciences, because everything positive in appearance is mere phenomenon; the phenomenon transcends its own appearance by becoming an object, turning into something positive; the negation of the phenomenon's negative appearance becomes the possible partial

knowledge of reality in itself [the identification of social and natural is a condition for the identification of everything as artificial, especially in cyborg evolution].

5.2.1.1.1.2.4.1.2.4. In Impossible Probability, priority is given to the Mean Deviation as the average of the absolute value of the possible differentials; the Mean Deviation does not alter the differentials by squaring or taking roots. The Mean Deviation preserves the proportionality of the differences themselves.

5.2.1.1.1.2.4.1.2.5. The sum of the absolute values of the normal Bias Levels equals Total Bias, and Total Bias divided by N—or Total Bias times the inverse of N—is the Mean Deviation. All bias is explained by chance and all chance by bias. The inverse of N is at once the ideal behavior by chance, and, in universes of infinite subjects or options, it is the theoretical probability of dispersion: theoretical probability of random occurrence, 1/N, and theoretical probability of bias, 1/N. In universes of limited options, as N tends toward the two-option paradigm (heads/tails, woman/man), the inverse of N tends to be the greatest possible Mean Deviation; therefore, in universes of limited options, the inverse of N is at the same time chance and the tendency toward the greatest sample bias.

5.2.1.1.1.2.4.1.2.6. The Average Bias Level explained by chance is the distribution of Total Bias by chance—the Mean Deviation, the probability of empirical dispersion within the sample. The Mean Deviation is the probability of the sample's empirical dispersion, caused by the sampling itself and by the sample's magnitude. In universes of infinite subjects or options, normally, the greater N is, the smaller the Mean Deviation. In universes of limited options—except in studies that in a first or second measurement require the sample to tend toward zeros—the greater the amount of raw scores or frequencies, normally, the smaller the Mean Deviation. Thus, the Mean Deviation is a direct product of the sampling procedure and of the sample's magnitude, whether it is an N sample in universes of infinite subjects or options, or a sample of raw scores or frequencies in universes of limited options. If a larger sample yields greater representativeness, and greater representativeness yields a smaller theoretical random bias in the sample, and a smaller theoretical random bias yields a smaller empirical chance-bias, then the empirical sample dispersion is inversely proportional to the sample—although one must qualify that the objective assessment of what

counts as error or reliability in terms of empirical sample dispersion will depend on the object of study. Within Impossible Probability we must distinguish three different modalities of empirical error or reliability: the first two for normal dispersion models whose dispersion ranges between zero and maximum, and a third modality for omega models. The First Modality of Empirical Error, in normal models of equal opportunity, equals sample dispersion over the maximum sample dispersion; therefore the First Modality of Empirical Reliability is one minus that quotient. The Second Modality of Empirical Reliability—if the model tends toward the maximum possible dispersion—will be sample dispersion over maximum sample dispersion; therefore the Second Modality of Empirical Error is one minus that quotient. The Third Modality of Empirical Error will be for omega models, when in an N greater than two there is an ideal number of subjects or options between two and N minus one; it will equal the quotient, relative to the ideal dispersion, of the absolute value of the difference between the empirical dispersion and the ideal dispersion (the ideal defined by the omega model). The Third Modality of Empirical Reliability is then one minus that quotient.

5.2.1.1.1.2.4.1.2.7. The sample's chance-bias is the behavior of chance within the sample. In studies of bias, the object of study is to validate the empirical hypothesis that tends to be true, insofar as it is shown that the bias of the empirical hypothesis is a rational or sufficient tendency; if the tendency is sufficient or rational, the policy of science will accept the hypothesis, transforming the empirical hypothesis into a truly rational hypothesis about objective reality. In studies of equal opportunity, the aim is to validate those hypotheses whose object is to show behavior tending toward the theoretical probability; if the tendency is sufficient or rational, the policy of science accepts the hypothesis, and it becomes a rational hypothesis that tends to be true within the objective reality of science. Politically accepting a hypothesis—so that it becomes part of the objective reality of science—means accepting the possible error that the objective reality of science is not entirely isomorphic: the objective reality of science is isomorphic to material reality only within the margin of error that the policy of science is willing to accept. Accepting a margin of error implies an empirical skepticism, or rational doubt, proportional to that margin of error—that is, accepting a margin of causal indeterminism within the apparent causality of the real. The progress

of science is the progress in scientific knowledge of reality in itself. *Progress is progress because knowledge is provisional*; the scientific models that replace previous ones will formulate deeper and superior hypotheses about what happens, within the ideological policy their scientific policy develops—each ideological model defines reality politically in a different way. *Every definition of reality is a political definition*. The political is to decide; to decide is to accept errors; to accept errors is to accept the provisional nature of what is decided. The provisional nature of scientific knowledge entails the provisional nature of political decision. But the provisional nature of political decision does not justify the provisional nature of moral principles*: the moral principle of science must be, universally, truth* [pure truth for Gaia, Mother]. The only universal moral maxim for science is that science must rationally tend toward truth. However, not even this universal maxim is absolute; it is relative, because each different ideological model will have a different morality, and for each different morality, truth is different. Only when the progress of science and humanity reaches the higher levels of history, and contemporary ideologies are surpassed by a science of permanent revolution—only then, perhaps, will the scientific moral principle of truth be the absolute universal moral maxim of science [the reason for everything, the solution of the matrix]. Until then, morality is relative to ideologies, and ideologies are relative to the interests they defend.

5.2.1.1.1.2.4.1.2.8. If the sample is not truly representative of the universe, then the sample's chance-bias is not representative of the universe, and therefore the chance-bias would be random error stemming from the sample's magnitude. If the sample is not sufficiently representative, the sample's chance-bias is the effect of error. Politically, it is unacceptable to accept an error that has been scientifically demonstrated to be an error in itself; policy must accept only what can be scientifically demonstrated within the margin of error. Whatever exceeds the margin of error lacks sufficient reliability; political morality must accept only truth—true, genuinely reliable truth—within the margin of error one is willing to accept.

5.2.1.1.1.2.4.1.2.9. The error of statistical inference is the error of attributing to the *universe behavior* [Astro ecology] that is actually caused by a sampling representativeness error, or because critical reason accepted an insufficiently

demanding margin of error. Reliable statistical inference is that which derives from representative samples under a sufficiently demanding, moral critical reason.

5.2.1.1.1.2.4.2. The control of sampling representativeness in any kind of universe

5.2.1.1.1.2.4.2.1. There are two ways to ensure that the sample is representative of the universe when sampling is necessary: by randomly selecting the sample, and by controlling that the sample is sufficiently representative. Samples may be the N sample in universes of infinite subjects or options, or the sample of raw scores or frequencies in universes of limited options.

5.2.1.1.1.2.4.2.2. Random selection of the universe, in any kind of universe—of infinite subjects or options, or of limited options

5.2.1.1.1.2.4.2.2.1. If we do not select a sample from the universe at random when sampling is necessary, the chance-bias measured by the sample will be the bias produced by error in the sampling technique.

5.2.1.1.1.2.4.2.2.2. If we select the sample at random, then the sample of biases is, in theory, representative of the *universe's chance behavior*—behavior not due to error in the technique itself.

5.2.1.1.1.2.4.2.2.3. Random selection of the sample normally guarantees the reliability of idea tests, ensuring that the sample's measured chance-bias is not caused by an error in the selection technique. A selection error would invalidate the contrast of ideas and render the sample's results unreliable. The only way to guarantee normal results in the sample is for the sample to be drawn at random from the universe, in cases where random selection is normally necessary. If an entire population is studied, of course, random sampling is not necessary.

5.2.1.1.1.2.4.2.2.4. Random selection is selection carried out independently of any factor or criterion other than pure chance. If, from a universe, we draw a series of subjects or options at random, or we select raw scores or frequencies at random, the random extraction will ensure normal sample results.

5.2.1.1.1.2.4.2.2.5. In a randomly drawn sample, hypothetically the bias is randomly reliable for contrasting ideas, because if there is behavior due to a sampling

representativeness error, the behavior of the error is not representative of a normal sample trend.

5.2.1.1.1.2.4.2.2.6. Only those empirical hypotheses are rationally acceptable whose bias- or chance-based propositions, as accepted by critical reason, are due to samples that are representative of the true universal trends.

5.2.1.1.1.2.4.2.3. Error and bias in N samples from universes of infinite subjects or options

5.2.1.1.1.2.4.2.3.1. The sample will be more representative of a universe of infinite subjects or options the larger the N. In addition to random sampling, controlling the number of subjects or options that make up the sample is a way to control sampling error—and thus control error in the sampling selection.

5.2.1.1.1.2.4.2.3.2. The greater the number of subjects or options in an N-sample, the greater the sample's representativeness, since the sample will include a larger number of distinct cases from the universe or population.

5.2.1.1.1.2.4.2.3.3. In an infinite universe, each subject or option is identified by some kind of differential behavior—an identifying bias of that subject's or option's individual chance. The *universe* is characterized by having a *universal identity* insofar as there is a characteristic bias of the universe. The trend of the universal bias—provided the sample is sufficiently representative—equals the distribution of Total Bias by chance, the Mean Deviation or Standard Deviation, within the margin of sampling-representativeness error.

5.2.1.1.1.2.4.2.3.4. The greater the number of subjects or options in the sample, the greater the sample's representativeness, because the sample more objectively represents the universe's possible diversity; such a sample would therefore be representative. Hence, the larger the number of subjects or options in the sample, the greater the likelihood that the sample adequately represents the universe's normal diversity.

5.2.1.1.1.2.4.2.3.5. The smaller the number of subjects or options in the sample, the lower the sample's representativeness, because the sample includes fewer subjects or

options representing the *universe's diversity*. Thus, the bias obtained in the sample may be the bias of the subjects or options in that particular sample and not necessarily a bias that represents reliably moral behavior.

5.2.1.1.1.2.4.2.3.6. It is a statistical error to attribute to a universe a bias extracted from a sample that does not morally represent that universe.

5.2.1.1.1.2.4.2.3.7. The success of the research lies in the policy of science deciding to accept only that bias or chance behavior which is sufficient or rational and representative—provided the sample is sufficiently or rationally representative. Only then is statistical inference possible; sufficient or rational sample representativeness is the condition of possibility for inferring the decision of critical reason.

5.2.1.1.1.2.4.2.3.8. If a greater number of subjects or options in the sample yields a smaller representativeness error in the statistical decision, and a smaller number yields a larger one, then the sampling representativeness error is a variable inversely proportional to the number of subjects or options in the sample, in universes of subjects or options.

5.2.1.1.1.2.4.2.3.9. If the error is inversely proportional to the number of subjects or options, in universes of subjects or options—i.e., the sampling representativeness error is inversely proportional to the N subjects or options of the sample—then the sampling representativeness error is the inverse of N. If the representativeness error is the inverse of N, then the inverse of N, 1/N, equals the error in the statistical decision due to sampling-representativeness bias. Therefore the inverse of N is the probability of error in statistical inference caused by sampling-representativeness error.

1/N = probability of sampling-representativeness error in universes of infinite subjects or options

5.2.1.2. The sample in universes of limited options: the sample of raw scores or frequencies “∑xi”

5.2.1.2.1. In a sample of N options—for example, if I toss a coin and it is heads or tails, then N equals heads or tails; or if N candidates run in an election, the N-sample will be the number N of candidates. In a sample limited to the options that make up the N options, to study the probability of sampling-representativeness error, as well as the theoretical probability of dispersion, in universes of limited options, the sample used to calculate theoretical dispersion and sampling-representativeness error will be the sample of raw scores or frequencies, ∑xi, the sum of the frequencies or raw scores.

5.2.1.2.1.1. If in a sample of N options—for example, coin tossing with two options, N = 2, heads or tails—the number of coin tosses will be the total number that forms the sample of raw scores or frequencies. If we say there were so many tails and so many heads, after so many tosses, then the total number of tosses will be the total number of the toss sample, the total sum of all tosses, whether heads or tails; the sum of the frequency of heads and the frequency of tails equals the total frequency, which is the sample of raw scores or frequencies, ∑xi. The more coin tosses are made, the more representative the sample of raw scores or frequencies, hence the smaller the sampling-representativeness error; the fewer the tosses, the less representative the sample and the greater the error. In democratic elections, the more voters there are, the more representative the results; the fewer voters, the less representative. That is, the greater the total sum of raw scores or frequencies of all candidates, the smaller the sampling-representativeness error; the smaller the sum, the greater the error. Therefore, in universes of limited options, the probability of sampling-representativeness error is inversely proportional to the sample of raw scores or frequencies—except when the ideal is a sample of zeros.

5.2.1.2.2. In a sample of N limited options, the N options are the options that bound the universe. In a universe limited to N possible options, the N-option sample will have a sample ∑xi of raw scores or frequencies. A possible study of N limited options may be a study of discrete categories.

5.2.1.2.2.1. In studies of discrete categories, what is normally done is to transform a universe of infinite subjects or options into universes of limited options [i.e. transformation of factors as subjects into categories as options] —options limited to the discrete categories—provided the number of discrete categories is significant and representative, as will be explained later.

5.2.1.2.2.2. Suppose we transform a universe of infinite subjects or options into a study of limited options—into a limited number of possible discrete categories—turning a truly infinite universe into a universe limited by options, options limited to a finite set of discrete categories. Suppose we have a sample of supernovae with different energy radiation. In a study of energy frequencies radiated by supernovae, if we group all supernovae into categories, then each category becomes its own option, and the frequency of each category would be that category's raw score. Suppose we have a sample of supernovae and classify them into discrete categories according to levels of radiated energy; then each discrete ***energy-radiation*** category would become an option, giving N options—discrete categories of radiated energy—and the frequency of each discrete category would become that category's raw score, each discrete energy-radiation category being a categorical option, with the frequency equal to the number of supernovae whose radiated energies fall in that category. In this case the N-category sample would be the N limited options, and the number of supernovae per category would be the frequency of each category; thus the sample $\sum x_i$ of raw scores or frequencies would equal the total number of supernovae. In a sample of N options, the frequency of each option can be interpreted as that option's raw score, and the total of frequencies or raw scores, $\sum x_i$, equals the sum of all frequencies or raw scores. In any case, the statistical results obtained from the discrete categories will only be representative if the supernova sample is itself representative. If a classification of supernovae by energy-radiation categories is made on a sufficiently representative sample of supernovae in the universe, then, necessarily, on a sufficiently representative sample the study's results will be theoretically reliable; that is, whether the hypothesis is accepted or rejected is a reliable decision because it is based on a representative sample of raw scores or frequencies. If a representative sample of supernovae requires a sufficient number of supernovae—and the sum $\sum x_i$ of raw scores or frequencies

across the discrete categories equals the total number of supernovae in the sample—then, logically, the greater the sum of raw scores or frequencies, the smaller the sampling-representativeness error of the frequency of supernovae per category, since the sample of supernovae is sufficiently representative; and the smaller the sum, the greater the representativeness error, since the sample is not sufficiently representative. Therefore, the inverse of the raw scores or frequencies is likewise the probability of sampling-representativeness error in universes limited to discrete categories—except when the ideal is a sample of zeros.

5.2.1.2.3. In universes of limited options, insofar as sample representativeness depends on the sample of raw scores or frequencies, the probability of sampling-representativeness error in universes of limited options will equal the inverse of the raw scores or frequencies; therefore Theoretical Reliability equals one minus the inverse of the raw scores or frequencies—except when the ideal is a sample of zeros, as explained in section nine.

$1/\sum x_i$ = probability of sampling-representativeness error in universes of limited options

$1 - 1/\sum x_i$ = Theoretical Reliability in universes of limited options

5.2.1.2.4. If, in universes of infinite subjects or options, over the N-sample the theoretical probability of dispersion equals the inverse of the N-sample, then in universes of limited options the theoretical probability of dispersion will equal the inverse of the raw scores or frequencies. Likewise—within the multifunctionality that, in universes of limited options, the inverse of the raw scores or frequencies also has—the inverse of the raw scores or frequencies in universes of limited options will also be the theoretical probability of dispersion: the fewer the coin tosses, the fewer the voters in an election, the fewer the supernovae in a study of discrete categories of energy radiation by supernovae—in general, the smaller the amount of raw scores or frequencies, the greater the theoretical dispersion; therefore, the greater the amount of

raw scores or frequencies, the smaller the theoretical dispersion.

$1/\sum x_i$ = theoretical probability of dispersion in universes of limited options

5.3. The raw score

5.3.1. Raw score and frequency

5.3.1.1. In a model with N options, in universes of limited options, the frequency of each option is interpreted as that option's raw score.

5.3.1.2. In a model with N subjects, in universes of infinite subjects or options, the magnitude of that subject's quality is interpreted as a raw score.

5.3.2. A raw score is the quantification of a quality; frequency is the number of times an event occurs. In sum, the raw score or frequency is the magnitude of an event—the quantitative value of the occurrence.

5.3.3. In Impossible Probability, the raw score or frequency of a subject or option will be represented as "xi," where "x" denotes the raw score or frequency and "i" denotes the subject or option, so that:

x_i = raw score or frequency of any subject or option

x_1 = raw score or frequency of the first subject or option

x_2 = raw score or frequency of the second subject or option

x_3 = raw score or frequency of the third subject or option

x_n = raw score or frequency of a given subject or option

5.3.4. The sum of the raw scores or frequencies of all subjects or options equals the sample of raw scores or frequencies, and is written mathematically as "∑xi," where the symbol "∑" means summation and "xi" denotes raw scores or frequencies. Thus "∑xi" represents the summation of the raw scores or frequencies—the sample of raw scores or frequencies.

5.4. Empirical probability

5.4.1. Empirical probability is a probability—a rational number, a quotient—in which a part is divided by a whole.

5.4.2. The raw score or frequency of a subject or option is only one part of the sample of raw scores or frequencies.

5.4.3. The summation of raw scores or frequencies for all subjects or options equals the sample of raw scores or frequencies.

5.4.3.1. In those specific cases where measuring a quality for a sample of subjects or options in infinite universes requires scales that use negative and positive integers (or only negatives), the summation is taken over the absolute value of the raw scores (without sign). Likewise, for computing each subject's or option's empirical probability, use the absolute value of the raw score divided by the summation of all absolute values of all raw scores. Since the use of measurement scales with negative integers is localized, in general we will simply say that the summation of raw scores or frequencies equals the sum of all raw scores or frequencies, and empirical probability equals raw score or frequency over the summation—bearing in mind that for negative integers, the sign is ignored both in the summation and in the calculation of empirical probability.

5.4.4. The quotient of a part over the whole gives the proportion of the part relative to the whole.

5.4.5. Probability, as a measurement of possibilities, has predictive value.

5.4.6. Empirical probability is the proportion of the individual part to the sample total; empirical probability is essential in the Second Method of Impossible Probability (October 16, 2002).

5.4.7. Empirical probability equals the quotient of the raw score or frequency of a subject or option divided by the sample of raw scores or frequencies, always using absolute values, and is written mathematically as follows. The symbol “p(xi)” will always denote empirical probability:

$p(xi) = xi : \sum xi$

p(xi) = empirical probability

5.4.8. The creation (October 16, 2002) of the Second Method of Impossible Probability starts from a definition of empirical probability. The Second Method’s empirical probability resembles, in the First Method, relative frequency—but with the difference that in classical statistics (which Impossible Probability calls the First Method), relative frequency is only the individual frequency over total frequency. In Impossible Probability, empirical probability is the raw score or frequency of a subject or option over the sample of raw scores or frequencies for the entire sample.

5.4.9. In the First Method—classical statistics—the possible sample is recognized as the sample of frequencies, where:

First Method → $\sum fi = N$ = summation of frequencies

5.4.10. In the Second Method (October 16, 2002), precisely, N is the number of subjects or options, and "∑xi" is the sample of raw scores or frequencies:

Second Method → ∑xi = sample or summation of raw scores or frequencies

Second Method → N = number of subjects or options

5.4.11. The main difference between the Second Method and the First Method lies in how N is evaluated and how the sample of raw scores or frequencies, ∑xi, is evaluated. For the Second Method, N is the sample of subjects or options, and ∑xi is the sample of raw scores or frequencies.

5.4.12. The chief property of empirical probability is that the summation of the empirical probabilities equals one:

$$\sum p(xi) = 1$$

5.4.13. Since the summation of empirical probabilities equals one, and in the Second Method N is taken as the sample of N subjects or options, it follows that the arithmetic mean of the empirical probabilities equals the inverse of N, 1/N:

$$\sum p(xi) : N = 1/N = \text{inverse of } N$$

5.4.14. Thus we reach one of the fundamental properties of the inverse of N in the Second Method. The inverse of N is at once the probability of sampling-representativeness error and the theoretical dispersion/deviation probability in samples of N subjects or options, in universes of infinite subjects or options. And the inverse of N is, at the same time, the arithmetic mean of the empirical probabilities and the theoretical probability of occurrence by chance under equal opportunities, in any

kind of universe—both infinite subjects/options and limited options. The inverse of N has further functions, which I will explain later.

5.4.15. Properties of empirical probability

5.4.15.1. The Second Method's empirical probability is a redefinition of the First Method's concept of relative frequency (classical statistics). In fact, the essence of Impossible Probability is, essentially, to redefine the fundamental concepts of statistics to move toward new mathematical notions that permit new transcendental equations. In the Second Method, empirical probability is the proportion between a subject's or option's raw score or frequency and the sample of raw scores or frequencies of all subjects or options—that is, the empirical probability of a subject or option is the proportion of the individual part to the sample whole; in this sense, multiplied by 100 it is a percentage. In any case, empirical probability is the subject's or option's individual empirical trend within the sample.

5.4.15.2. As a probability, empirical probability is a predictive term for a subject's or option's future behavior. According to that subject's or option's probability, we can make future predictions based on the observed trend; thus empirical probability serves an individual predictive function.

5.4.15.3. Empirical probability is a proportion between whole and part, where the whole is the sum of the parts (the parts being raw scores or frequencies). In this sense, empirical probability also serves a descriptive function.

5.4.15.4. In the empirical probability model, a change in any part changes the whole—because if one part changes, then the whole changes.

5.4.15.4.1. If one part decreases, the whole decreases [if one option or subject decreases the sum decreases], If, in a second measurement, a given subject or option obtains a lower raw score or frequency, then the summation of raw scores or frequencies in the second measurement will decrease in proportion to that subject's or option's decrease. Provided all other subjects or options keep their raw scores or frequencies unchanged, while the total summation decreases due to at least one subject's or option's decrease, logically—even if all other subjects or options keep their

raw scores or frequencies unchanged compared with the first measurement—all the other subjects or options will see their empirical probabilities increase in the second measurement. Although their raw scores or frequencies remain the same, their proportional value rises as the total summation falls because at least one subject or option decreased; the subject or option whose raw score or frequency decreased will normally tend to have a lower empirical probability in the second measurement compared to the first.

5.4.15.4.2. If one part increases, the whole increases [the summation increases]. If, in a second measurement, a given subject or option obtains a higher raw score or frequency, then the summation of raw scores or frequencies will increase in proportion to that subject's or option's increase. If one subject or option increases its raw score or frequency—thereby increasing the sample of raw scores or frequencies—in proportion to that increase, then all other subjects or options, even if their raw scores or frequencies remain unchanged, will see their empirical probability decrease in the second measurement. Although their raw scores or frequencies remain the same, as the total summation grows their proportional value shrinks, while normally only the empirical probability of that one subject or option that increased its raw score or frequency will rise.

5.4.15.4.3. Any minimal variation, however insignificant, in the raw score or frequency of a single subject or option can produce changes in the empirical probabilities of all subjects or options.

5.4.15.4.4. Empirical probability is dialectical, because it grasps what Hegel already defined as a feature of dialectic: any minimal insignificance can become a determining factor—everything insignificant can become significant, and everything significant can become insignificant; everything non-determinant is potentially determinant, and everything determinant may in fact be non-determinant. In Impossible Probability, statistics is the history of chance; all chance is bias, all bias is chance—within the margin of error we are willing to accept. Chance determines everything, and everything determines chance. Science is the way to intervene in chance, accepting a margin of error—that is, accepting a margin of chance, a margin of determining indeterminacy, a margin of empirical skepticism or rational doubt; in any case, rational critique—

accepting the inevitable consequences of science for the possible future. Any minimal variation in a single subject or option will produce changes in the empirical probability of all other subjects or options. An individual change can change everything; a simple change in the individual history of a subject or option can change the whole history. A minimal, insignificant variation in one subject or option can produce determining transformations in the entire system. Statistical probability is the study of what happens in chance and history, where a single event can be enough to change history.

5.4.15.4.5. The only way the raw scores or frequencies of the subjects or options can change from a first to a second measurement without changing each subject's or option's empirical probability is if all subjects or options change at the same time without altering their proportional differences—i.e., they all increase or decrease their raw score or frequency simultaneously while preserving the original proportional differences between their part and the whole. Only if everyone changes at the same time, yet without varying their proportions between their parts and the whole, need the probabilities remain unchanged, provided the changes respect the individual proportionalities. However, it is enough for just one individual to change for everything to change.

5.4.15.4.6. In Impossible Probability, the true ideal change is change toward the realization of the ideal: under equality of opportunity the ideal is the tendency of bias toward zero; in the study of positive bias the ideal is the increase of the positive bias of the ideal option (or of the ideal options in omega models); in the study of negative bias the ideal is the increase of negative bias. The ideal object of study depends on the ideological definition of the ideals; each model of scientific policy will tend to achieve the ideals of its ideological politics. Morality is a universal maxim relative to political ideology, which will determine how demanding the margin of error must be to accept its causal hypotheses. Science, insofar as it is ideological, is relative, provisional; if there is to be genuine moral progress, it should be toward the moral truth of the universe—the objective reality in itself—even if, for the moment, it is only partial in science.

5.5. The inverse of N

5.5.1. The inverse of N equals the quotient obtained by dividing the unit by N—a rational quotient resulting from dividing the first factor, the unit, by the second factor, N.

5.5.2. Definition of N

5.5.2.1. N equals the number of subjects or options in the sample. In a study of N possible options, in universes of limited options, the N possible options are N. In a study of N subjects or options, N equals those N subjects or options.

5.5.2.2. N is thus formed from N subjects or N options, where each subject or option will be represented mathematically by the symbol “ni,” in which “n” indicates an nth position in the sample N, and “i” refers to the subject or option; thus:

ni = any subject or option

n1 = first subject or option

n2 = second subject or option

n3 = third subject or option

5.5.3. The unit

5.5.3.1. Mathematical characteristics of the unit

5.5.3.1.1. The unit is the first natural number and the first prime number, and it is the number upon whose nth addition the other natural numbers are formed.

5.5.3.1.2. Dividing the unit by its parts yields a number less than the unit but not less than zero—a number between zero and one, a purely decimal number with no integer part, a positive rational number less than one, characterized by a zero followed by a comma and then the decimals.

5.5.3.1.3. There are infinitely many purely decimal rational numbers, and there are also infinitely many rational numbers that have an integer part followed by the decimal comma and decimal digits—though within this latter type (integer part plus decimals) there may also be irrational numbers.

5.5.3.1.4. The possible infinity of the natural numbers is due to the fact that, from the unit to infinity, the possible positive numbers are infinite.

5.5.3.1.5. The possible infinity of the integers is due to the fact that, from minus infinity to plus infinity, the possible numbers are infinite; it is within the integers that absolute zero appears—the mathematical value of nothingness.

5.5.3.1.6. Rational numbers are formed as quotients of numbers and have the particular feature of being able to be decimal numbers.

5.5.3.1.7. The infinitely many possible decimal numbers arise because the unit is infinitely subdivisible; between any number and any other number there are infinitely many rational numbers [here the question is the division between the unit and the infinite given an infinite sequence of zeros, is rational or irrational? Impossible Probability=PI].

5.5.3.1.8. The infinite set of decimal numbers is not only infinite in quantity; individual decimals themselves may be finite (i.e., the decimal sequence has an end) or infinite (i.e., the decimal sequence has no end).

5.5.3.1.9. Note that among integers followed by decimals there may be irrational numbers, and infinitely many of them, where the end of the decimal sequence is unknown and believed to be infinite; such is the case with the number *pi*.

5.5.3.2. Unit, number, and probability

5.5.3.2.1. Any whole is a total whole that integrates all the possible parts of that whole.

5.5.3.2.2. Any whole is, above all, one in itself—i.e., it is a one, the representation, as a one, of the unit.

5.5.3.2.3. A whole, whatever it may be among all possible wholes—one among them, a singular whole within all the singular wholes—is a whole; insofar as it is a total one, a

whole, it is a singular total. The total unit of any whole is the singular unit of that singular total whole; the singular unit of a possible whole, among all possibles, is the unit singular in itself.

5.5.3.2.4. A singular unit is a singular unity; insofar as it is singular, it is in itself a single, unique unit. A singular whole is the in-itself unity of that singular for-itself, which expresses itself outside itself as the totality of all its parts integrated into that singular unity, distinguishing it from any other possible unit.

5.5.3.2.5. A singular unit can be divided into its parts, and every unit can be subdivided infinitely into all its infinite parts. Insofar as the unit is infinitely divisible, the infinite totality of the singular unit constitutes the unit; and the singular unit, as the unit of any possible total (infinitely divisible), is the unit of a possible infinity. The unit of the infinite is the unit in itself. Everything infinite is infinite because it can be infinitely subdivided; the unit in itself, being infinitely divisible, is the unit of an infinite.

5.5.3.2.6. A particular is a particular—that is, a particular **could be me, could be the computer**; a particular could be the heads or tails of a coin; a particular could be the male sex in particular or the female sex in particular, or a particular political option; a particular could be each animal that composes the fauna of an ecosystem; a particular **could be an asteroid, a comet, a galaxy among the possible galaxies**; a particular could even be **one universe among the possible universes**; a particular could be a particular **photon**, a particular electron, a particular neutron, a particular quark. A particular is any possible object or subject, or any possible subject or option.

5.5.3.2.7. A particular may be object or subject, or subject and object at the same time—just as a particular may be an option or a subject, or subject and option at the same time. A particular, for itself, is a unit**: I am my own unit of all my being; every being is a being insofar as it is the unit of all its being. A galaxy is the unit of all the elements of the galaxy; each universe among the possible universes is a universe composed of all the elements of that universe; a photon** is a photon in itself; an electron, neutron, or quark is in itself its own unit of all the elements that form it. The unit of being is the being-in-itself, recognizing itself as for-itself; outside itself, being is a unit different from any other possible unit.

5.5.3.2.8. Every particular is characterized by its particular qualities, and every particular quality is characterized by a measurability—measurement, a numerical expression of that quality—inasmuch as all that is qualitative becomes quantitative and vice versa. Subjective qualitative reality is transformed into objective quantitative ideas. The contrast between idea and reality is the contrast between the scientific, objective idea and the physiological idea—brain substance of empirical origin—the subjective idea of reality. Every contrast is a contrast of ideas concerning the quantitative dimensions of the qualities of reality—the contrast between objective reality and perceived subjective reality.

5.5.3.2.9. Every quality is a quality of a subject or option, and every quality in itself is likewise a particular for itself—as a particular quality of a particular subject or option—such that, when the quality is expressed numerically, it equals the total sum of singular units in the measurement, according to a singular unit of measure, by which the quality is quantitatively displayed as proportioned—measured—against the singular unit of measure. The result of the measurement is known as the raw score. The raw score of a particular quality equals the sum of the singular units in which that particular quality is expressed in the measurement; the raw score is the intensity of the quality—the magnitude of that particular quality.

5.5.3.2.10. The raw score of a particular quality equals the sum of the singular units of measure in which the quality of the particular is expressed; and the total sum of absolutely all raw scores is the sample of measurements of the particular quality across the whole sample. The total sum of the sample's raw scores is the sample of raw scores. Frequency, as the number of times something happens, can be interpreted or understood as a raw score; and the raw score can be interpreted or understood as a frequency. Frequency and raw score are identical because they are opposites. A frequency is the number of times a quality is expressed; the number of times the quality is expressed is a quantitative magnitude of that quality—i.e., frequency can be understood as the raw score of the magnitude of the frequency of the quality, the intensity of repetition of an occurrence. A raw score, as the sum of the singular units in which a subject's or option's quality is expressed in the measurement, is the magnitude of a particular quality of the subject or option. Thus, if frequency is magnitude and raw

score is magnitude, frequency and raw score are identical insofar as they are different expressions of the magnitude of the quality: the raw score measures the magnitude of the intensity of the quality itself, and frequency measures the magnitude of the intensity of the occurrence of the quality.

5.5.3.2.11. To know what raw score or frequency, in theory—by chance, ideally—each particular subject or option in that sample should theoretically, by chance, ideally have under normal conditions, as a function of the sum of all raw scores or frequencies, one divides the sample of raw scores or frequencies by the number of subjects or options in that universe from which that universal sum was obtained; that is, one computes the arithmetic mean of the raw scores or frequencies [First Method].

5.5.3.2.12. The arithmetic mean of the raw scores or frequencies is the theoretical raw score or theoretical frequency that, under equality of opportunity—by chance—should correspond to each subject or option in that sample. The arithmetic mean measures what the normal theoretical behavior by chance—the ideal—should be under conditions of equal opportunity, under normal conditions: chance. Normally, naturally, any random distribution tends to distribute itself, by mere chance, around the arithmetic mean—the statistical norm, the highest point of the normal curve. Normally, naturally, the greater the number of subjects or options, the more the sample tends to distribute itself by chance around the statistical norm.

5.5.3.2.13. The total number of N subjects or N options that make up a sample is equal to the number N of subjects or N options that constitute that sample. That given sample may be equal to the number of options in universes of limited options, or it may be equal to the number of subjects or options randomly selected in universes of infinite subjects or options, or it may be equal to the number of subjects or options in a given population.

5.5.3.2.14. Thus, N equals the number of either N subjects or N options; in any kind of universe, N is the number of options or subjects, and the arithmetic mean of the raw scores or frequencies equals "$\sum xi : N$." The arithmetic mean of the raw scores or frequencies is the theoretical raw score or theoretical frequency that, in theory, should correspond to each subject or option under equal opportunities. Equality of opportunity is equality of occurrence by chance.

5.5.3.2.15. Under equal opportunities, all subjects or options should have the same raw score or frequency. If all human beings had the same intelligence quotient, then all humans would obtain the same raw score on an intelligence test. If all stars had the same amount of hydrogen and helium, then all stars would have the same amount of energy, mass, and volume. If all planets had the same mass, then all planets would have the same gravity, and spacetime curvature would be the same. In conclusion: under equal opportunities—equal conditions for occurrence—all would have the same raw score or frequency. Equality of conditions for occurrence would make all raw scores equal to the mean of the raw scores or frequencies; therefore, in a model with such characteristics, any random event would be equal to any other event. The fact that any random event is equal to any other event would imply that all events are equal to the norm, the arithmetic mean—empirical deviation or dispersion zero, Mean or Standard Deviation zero, bias zero. In empirical reality it is normal for any random sample to tend to distribute around the arithmetic mean, with a normal empirical oscillation error about the central tendency equal to the Mean or Standard Deviation; typically, empirical reality tends to be normal within a margin of Standard Deviation. And the larger the sample size—whether an N-sample in universes of infinite subjects or options, or a sample of raw scores or frequencies in universes of limited options—the greater the tendency toward normality of empirical reality; the more the sample increases, the greater the concentration of subjects or options around the arithmetic mean, the statistical norm. Normally, all subjects or options tend to equal the arithmetic mean, within the plus-or-minus one of the normal curve—the normal empirical oscillation error about the central tendency. The larger the sample, the smaller the bias; the smaller the sample, the greater the bias. Normally, bias is reduced as the sample tends to increase; empirical random bias is inversely proportional to the sample. If the normal tendency is that a larger sample yields greater random ordering around the arithmetic mean, then in an infinite tendency to increase the sample to infinity, in the infinite the normal would be the infinite identity of the infinite with the norm; what is normal in the infinite is to be normal—the arithmetic mean. In the infinite, deviation or dispersion is zero; in the infinite, error is zero; the infinite must be normal—ideally, to be normal. Being normal in a truly infinite universe of subjects or options in all its dimensions—and thus in infinite time—would then imply empirical probability

equal to the inverse of N, i.e., zero, Impossible Probability, if it truly is a universe that is infinite in all its dimensions.

5.5.3.2.16. Under equal conditions of occurrence by chance—the free will of phenomena—where all subjects or options have the same raw score or frequency, the differential between raw score and arithmetic mean (the differential score, the bias) would be zero. Thus every subject or option would behave according to the ideal theoretical behavior under equality of opportunity: the ideal of equality in free will.

5.5.3.2.17. The arithmetic mean indicates what the theoretical random behavior should be, under equality of opportunity within the free will of reality—under equal conditions—for every subject or option, insofar as it is normal that, by chance, every subject or option tends in the infinite to distribute around the arithmetic mean, typically within plus or minus one Standard Deviation of error on the normal curve—normal empirical oscillation error about the central tendency.

5.5.3.2.18. Empirical probability equals the quotient of the particular raw score or frequency of the subject or option divided by the sum of all raw scores or frequencies in the sample.

5.5.3.2.19. Empirical probability times 100 is a descriptive percentage of reality, and empirical probability as a probability is a predictive value.

5.5.3.2.20. The sum of all empirical probabilities equals one.

5.5.3.2.21. The arithmetic mean of the empirical probabilities equals the sum of all empirical probabilities—that is, one—divided by N; that is, the arithmetic mean of the empirical probabilities equals the inverse of N, 1/N.

5.5.3.2.22. The inverse of N, as the arithmetic mean of the empirical probabilities, equals the theoretical behavior by chance—the central tendency of the sample under equality—the theoretical freedom of free will in equality of opportunity. The Mean Deviation is, in fact, the bias of free will in practice—in empirical reality—not necessarily in accord with objective equality of opportunity.

5.5.3.2.23. In this way the inverse of N is both arithmetic mean and theoretical probability, insofar as the inverse of N equals the probability of occurrence under equal

conditions by chance—free will—and it will be symbolized either as “p(xt)” (theoretical probability), especially in the first phase, or directly as the inverse of N, 1/N, especially from the second phase on. Wherever the inverse of N, 1/N, appears in equations, it will appear without brackets or any other mathematical delimiter of that operation’s unit; whenever 1/N appears in an equation, it should be interpreted directly as the inverse of N and computed operationally as such, even if shown without brackets.

5.5.3.2.24. Theoretical probability, as a theory of chance—free will under equality of opportunity—is predictive of behavior in the absence of bias; it forecasts what behavior should be under conditions of chance.

5.5.3.2.25. The difference between empirical probability and theoretical probability equals the normal Level of Bias of a subject or option; it is called positive bias if the differential is positive, negative bias if the differential is negative.

5.5.3.2.26. The sum of all negative normal Levels of Bias equals the sum of all positive normal Levels of Bias. Thus, to compute Total Bias—the sum of normal Levels of Bias—one must sum the absolute values of the normal Levels of Bias, without sign. The Mean Deviation is either Total Bias divided by N (Average Level of Bias) or Total Bias times the inverse of N; the distribution of Total Bias by chance is, empirically, the behavior of chance in the real sample.

5.5.3.3. Synthesis of the inverse of N to this point (further functions of the inverse of N will be explained later)

5.5.3.3.1. So far we have seen that in universes of infinite subjects or options, the inverse of N, 1/N, is the probability of sampling representativeness error.

5.5.3.3.2. At the same time, the inverse of N, i.e., 1/N, equals the arithmetic mean of the empirical probabilities in any kind of sample of any kind of universe—whether of infinite subjects or options, or of limited options.

5.5.3.3.3. At the same time, the inverse of N, i.e., 1/N, equals the theoretical probability that everything occurs by chance, under equality of opportunity, in any kind of sample of any kind of universe—whether of infinite subjects or options, or of limited options.

5.5.3.3.4. Conclusion on the inverse of N so far: the inverse of N, 1/N, is a complex mathematical expression in Impossible Probability—multifunctional, capable of assuming different, opposing, and identical functions at the same time. It is simultaneously an arithmetic mean, the theoretical probability of behavior under equal conditions, and, in universes of infinite subjects or options, the probability of sampling representativeness error. Moreover, this complexity entails an even more complex feature: the inverse of N is the probability of theoretical dispersion or deviation—in universes of infinite subjects or options, the inverse of N is the theoretical probability of bias by chance, theoretical dispersion.

5.5.4. The inverse of N as theoretical dispersion probability—theoretical probability of sample bias—in universes of infinite subjects or options

5.5.4.1. Mean Deviation, Variance, and Standard Deviation

5.5.4.1.1. The First Method—the classical, conventional statistics—recognizes three dispersion statistics, namely: Mean Deviation, Variance, and Standard Deviation.

5.5.4.1.2. Mean Deviation

5.5.4.1.2.1. Mean Deviation equals the mean of the deviations—the central tendency of the deviations, the tendency of dispersion—where the possible deviations are the number of differential scores in a sample, the differential score being the difference between raw score or frequency and the arithmetic mean; in Mean Deviation this difference is taken in absolute value, and it is denoted by the capital letters “DM.”

5.5.4.1.2.2. Since taking account of the signs of the differential scores in their sum yields a result equal to zero—because of natural goodness—then, normally, the Mean Deviation only has effectiveness if one considers the absolute values of the differentials; that is, without taking the sign into account in the sum of the differentials.

5.5.4.1.2.3. In Impossible Probability, Mean Deviation will be a customary empirical dispersion statistic, where the sign of the resulting differential is not taken into account in the sums of the Levels of Bias, in order to average the sum of absolute values. Mean Deviation is itself the mean of the deviations—the central tendency of the sample’s empirical dispersion.

5.5.4.1.3. Variance

5.5.4.1.3.1. Classical or conventional statistics (the First Method) does take into account the signs in the sum of the differential scores; to neutralize the signs of the differentials it squares the differentials: any negative integer squared becomes a positive integer square. Variance is the average of all squared differential scores.

5.5.4.1.3.2. Variance is a squared dispersion statistic; it indicates the mean squared differential between raw scores or frequencies and the mean. But if we want to know the proportion between a differential score and the Variance, this is not mathematically possible—i.e., not meaningful—because the differential score is not squared while the Variance is. The only way to compare would be by the quotient of the squared differential score divided by the Variance. However, this expression is not commonly used; traditionally the Standard Score (z-score) is used, for which the square root of the variance is needed.

5.5.4.1.3.3. Variance is normally symbolized by "s^2."

5.5.4.1.4. Standard Deviation

5.5.4.1.4.1. Variance is the average of the squared differential scores; but since each differential itself is not squared—squaring is applied to the differential scores to nullify the sign's effect when summing for the Mean Deviation—one usually takes the square root of the variance, and the square root of the variance is the Standard Deviation, denoted by an uppercase "S."

5.5.4.1.4.2. The Standard Deviation is the square root of the mean of the squared deviations, and the ratio between a differential score and the Standard Deviation is known as the Standard Score (z-score), expressed as:

Standard Score =

$(xi - \overline{X}) : S$

5.5.4.1.4.3. The usual pattern of Standard Scores traces a Gaussian curve, a curve attributed to the mathematician Gauss. It is, in essence, an integral whose highest point on the vertical axis lies at the center (zero on the horizontal axis of the Cartesian plane). The norm is the arithmetic mean, and the Standard Score of the arithmetic mean is zero, since mean minus mean equals zero—therefore, the Standard Score of the mean equals zero. Within an error of plus or minus one, it is normal to tend toward the arithmetic mean on the normal curve; the larger the sample, the stronger the random tendency of Standard Scores toward the arithmetic mean. In essence, being "normal" is to tend by simple chance toward normality.

5.5.4.1.4.4. The Enlightenment idea that nature is good by nature rests on the fact that the "goodness" of nature is the mathematical goodness of chance: normally, by pure chance, all the normal negative bias of a sample is compensated by all its normal positive bias. By pure chance, the total magnitude of normal negative bias will normally be compensated by the total magnitude of normal positive biases. In philosophy this natural goodness—this inherent tendency of nature to offset the negative with the positive, i.e., the goodness of chance—is traditionally called **immanent justice**. Immanently, in the sample, the sum of all negative bias will always equal the sum of all positive bias; hence, to compute Total Bias (the sum of absolute values of Levels of Bias) one must sum the absolute values of the normal Levels of Bias—i.e., the absolute value of empirical probability minus theoretical probability. Otherwise, since positive bias tends to offset negative bias by natural chance, the result of the sum would always be zero. The dialectic of the positive and the negative is often called in philosophy the **dialectic of the immanent, or immanent dialectics**. Because nature normally tends

to offset every negative bias with an equal positive bias, the positive and the negative are, dialectically, normally identical—they are identical because they will always have the **same** normal magnitude: every positive bias will, immanently and normally, imply a negative bias of equal magnitude, by simple chance, and every negative bias will, immanently and normally, imply a positive bias of equal magnitude, by simple chance. **Natural law is the law of chance; natural goodness is the law of probability.** In a truly infinite universe in all its dimensions—a universe of infinite subjects or options in infinite time—at the infinite everything is zero, i.e., nothing; zero is the mathematical value of nothing. In the infinite, everything is nothing, and nothing is everything. The natural law is that everything is determined by chance. Even our most unquestionable scientific ideas, when the moment of truth arrives, will inevitably bear the reality of the accepted margins of error.

5.5.4.1.5. Conclusion on dispersion statistics in the First Method (classical or conventional statistics): normally, in the First Method, the Mean Deviation is little used because sign issues are central; the most common choice in classical statistics is the Standard Deviation.

5.5.4.2. Dispersion in the Second Method

5.5.4.2.1. In the Second Method, Mean Deviation ("DM") has priority because, by preserving the essence of bias intact, it is the most objective statistic for the central tendency of biases in the sample. For the Second Method, the basis of the entire study lies in the differential relationship between empirical probability and the inverse of N; the more exact this value, the more objective the descriptive or inferential statistics. In the Second Method the differential score is redefined as the normal Level of Bias, i.e., the difference between empirical probability and theoretical probability, where theoretical probability is always the inverse of N. The sum of the absolute values of the normal Levels of Bias is Total Bias, and Total Bias divided by N is the Average Level of Bias—or the distribution of Total Bias by chance, i.e., Total Bias times the inverse of N. Mean Deviation is the empirical randomness of the sample, or the normal empirical oscillation error about the central tendency (the inverse of N); Mean Deviation is the sample's normal empirical bias. Since the First Method prioritizes Standard Deviation, in the Second Method—whenever one must choose an empirical sample dispersion—

the scientific policy may choose Mean Deviation or Standard Deviation; this will be symbolized by “DMoS” (Mean Deviation or Standard Deviation), either being acceptable. For the Second Method, Mean Deviation is always preferable as the more objective value of the sample’s empirical dispersion, more faithful to the sample’s true differential essence.

5.5.4.2.2. In the Second Method, the Variance, s^2, is logically the average of the squared Levels of Bias.

5.5.4.2.3. The Standard Deviation, S, is the square root of the Variance.

5.5.4.2.4. “DMoS” will indicate that scientific policy may choose Mean Deviation or Standard Deviation; in light of the Second Method it is always preferable to use Mean Deviation as the more objective value of the sample’s empirical dispersion—the most faithful to the sample’s true differential essence.

5.5.4.3. Inverse of N and Mean or Standard Deviation in universes of infinite subjects or options

5.5.4.3.1. To simplify the explanations of the relationship between theoretical and empirical dispersion, I will first give the necessary explanations for universes of infinite subjects or options, and then, based on the conclusions, explain the same process of relationships for universes of limited options, where the probability of theoretical dispersion is the inverse of the raw scores or frequencies, $1/\sum xi$. Thus, beginning with the relationship between N and empirical or theoretical dispersion/deviation in universes of infinite subjects or options: among its many functions, in such universes the inverse of N is the probability of sampling representativeness error. In samples from universes of infinite subjects or options, the way to reduce representativeness error is to include as many different subjects or options as possible; as N increases, the sample better represents the universe. The larger the sample, the more it naturally normalizes around the arithmetic mean; that is, the larger N is, the more subjects or options will have scores near the mean—provided the ideal of equality *holds*. The *Gaussian* curve is nothing more than the integral explanation of how the population normally tends to distribute around the mean, with a typical normal error of plus or minus one under normal conditions.

5.5.4.3.2. The larger the sample N, the smaller the sample's representativeness error. The larger N is—under equal conditions of random occurrence—the smaller the possible differential among behaviors of the subjects or options in the sample; as the sample grows, it naturally normalizes at the arithmetic mean, the norm. Thus, the larger N is, the lower the probability of sampling representativeness error, and the higher the probability of normal behavior; hence, the larger N is, the less representativeness error and the less differential behavior—i.e., the larger N is, the less error and the less bias. Bias and error are inversely proportional to N; therefore every bias is an error and every error is a bias, because randomness produces error and error produces randomness. In short, randomness, error, and bias are three different yet dialectically identical logical moments—each in relation to the others, and all in relation to each. Inverse N is the theoretical probability of equality by chance and, in universes of infinite subjects or options, the probability of sampling representativeness error and the probability of theoretical deviation/dispersion—i.e., theoretical bias. If the probability of theoretical dispersion/deviation is the probability of theoretical bias (since every dispersion/deviation is bias), and if the inverse of N is the theoretical probability of equality of occurrence by chance (i.e., theoretical randomness), then at the same time the inverse of N is the theoretical probability of bias and the theoretical probability of chance: theoretical bias (1/N) and theoretical randomness (1/N) are identical—1/N—inasmuch as, theoretically, bias and randomness are identical within the accepted margin of error, that margin being the sampling representativeness error itself, 1/N. Thus, in universes of infinite subjects or options, the inverse of N, 1/N, is at once: randomness, bias, and error. Randomness, bias, and error are three logical moments of the same dialectical process—in universes of infinite subjects or options, the inverse of N. Equality and freedom are identical within the accepted margin of error—free will under equal opportunity, i.e., chance. The moral margin of error in accepting freedom is freedom itself; merely accepting freedom morally entails accepting the margin of error stemming from freedom itself. *Freedom and moral error are identical*; hence moral responsibility follows: *every act of freedom must be politically responsible*. 1/N = theoretical probability of equality by chance, theoretical bias probability, and probability of sampling representativeness error, in universes of infinite subjects or options.

5.5.4.3.3. As N increases, the Mean or Standard Deviation—the sample's empirical bias—approaches zero; thus, as N tends to infinity, the inverse of N tends to zero, and as the inverse of N tends to zero (with N increasing), the Average Level of Bias tends to zero.

If $(N \to \infty) \to 1/N \approx 0 \to DMoS \approx 0$

5.5.4.3.3.1. Suppose we work in a tendency toward infinity, on a universe of N subjects or options tending to infinity.

5.5.4.3.3.2. As N tends to infinity, empirical probabilities tend to zero.

5.5.4.3.3.3. As empirical probabilities tend to zero—Impossible Probability—normal Levels of Bias tend to zero.

5.5.4.3.3.4. As Levels of Bias tend to zero, the Mean or Standard Deviation tends to zero—Impossible Probability of bias.

5.5.4.3.3.5. As the Mean or Standard Deviation tends to zero, it tends toward the inverse of N to the same extent that the inverse of N tends to zero. If so, then:

If $(N \to \infty) \to (1/N \approx 0) \to (p(xi) \approx 0) \to (DMoS \approx 0)$

5.5.4.3.3.6. As N increases, the inverse of N and the empirical probability will tend to zero; that is, as N increases, the normal Level of Bias decreases, and as N increases, the Mean or Standard Deviation decreases—in other words, as N increases, biased behavior diminishes; as N increases, empirical behavior tends to equal theoretical behavior by chance.

5.5.4.3.3.7. If, as N increases, dispersion decreases, then as N increases the empirical probability tends to equal the inverse of N, that is:

If $(N \to \infty) \to (p(xi) \approx 1/N)$

5.5.4.3.3.8. Or, in other words: as N decreases, the empirical probability increases; as N decreases, the Level of Bias increases; as N decreases, the Mean or Standard Deviation increases.

5.5.4.3.3.9. The relationship between N and the Mean or Standard Deviation is inversely proportional.
5.5.4.3.3.9.1. The larger the N, the smaller the Mean or Standard Deviation.
5.5.4.3.3.9.2. The smaller the N, the larger the Mean or Standard Deviation.

5.5.4.3.3.10. The relationship between N and the Mean or Standard Deviation is inversely proportional to N, provided normal conditions hold; that is, the greater N is, the lower the probability of the Mean or Standard Deviation, and the smaller N is, the higher the probability of the Mean or Standard Deviation. The Mean or Standard Deviation is the probability of empirical dispersion. Under normal conditions, the probability of empirical dispersion decreases as N increases—that is, the Mean or Standard Deviation is smaller as N grows. And the probability of empirical dispersion increases as N decreases—that is, the smaller the N, the larger the Mean or Standard Deviation.

5.5.4.3.3.11. If, for larger N, the probability of the Mean or Standard Deviation is lower—that is, the Mean or Standard Deviation tends toward zero as N increases—then the trend of the inverse of N (the theoretical probability of dispersion) toward zero and the trend of the Mean or Standard Deviation toward zero, as N increases, are proportional. As N increases, the inverse of N tends toward zero in direct proportion to the trend toward zero of the Mean or Standard Deviation. When N increases, the theoretical probability of dispersion—i.e., theoretical bias, the inverse of N—tends to zero, and proportionally the probability of empirical dispersion, the Mean or Standard Deviation,

also tends to zero. If the Mean Deviation is the average of the Levels of Bias—the Average Level of Bias—then the Mean Deviation is the probability of sample empirical bias; therefore *the inverse of N, 1/N, is the probability of theoretical bias*, since the *inverse of N is the theoretical probability of dispersion or deviation*. The inverse of N is at once the theoretical probability of equality of occurrence by chance and the probability of theoretical dispersion or bias. In universes of infinite subjects or options, the inverse of N is at the same time the theoretical probability of behavior by chance and the probability of behavior by bias; the inverse of N is the theoretical probability of chance and of bias—bias and chance are identical within the accepted margin of error, namely the inverse of N as the probability of sampling representativeness error in universes of infinite subjects or options.

5.5.4.3.3.12. If, for smaller N, the probability of the Mean Deviation is greater, the probability of bias increases; and if, for smaller N, the inverse of N is larger, then as N decreases the theoretical probability of dispersion tends to increase; as N decreases, the theoretical probability of bias—the inverse of N—tends to increase. As N decreases, the theoretical probability of bias (the inverse of N) increases, and directly proportionally the probability of sample empirical dispersion—the Mean or Standard Deviation—also increases; that is, the probability of sample empirical bias increases.

5.5.4.3.3.13. If, as N increases, theoretical or empirical dispersion decreases—since as N increases the inverse of N is smaller and, proportionally, the Mean or Standard Deviation is smaller—or, conversely, if as N decreases theoretical or empirical dispersion increases—since a smaller N implies a larger inverse of N and, proportionally, a larger Mean or Standard Deviation—then, in any study of a sample's central tendency, the representativeness of the central tendency is directly proportional to N. As N increases, the normal theoretical oscillation error around the central tendency decreases, since if the inverse of N is theoretical dispersion and the normal theoretical dispersion around the central tendency is the inverse of N, then as N increases the theoretical oscillation error around the inverse of N as central tendency tends to decrease; at the same time, as N increases, the normal empirical oscillation error around the central tendency decreases; the oscillation of the inverse of N relative to the Mean or Standard Deviation will be smaller the larger N is, and conversely, the

smaller N is, the greater the theoretical error (inverse of N) or empirical error (Mean or Standard Deviation) of normal oscillation around the central tendency (the inverse of N).

5.5.4.3.3.14. The representativeness of the central tendency of any sample is thus directly proportional to the magnitude N, in universes of infinite subjects or options, insofar as dispersion—whether theoretical (inverse of N) or empirical (Mean or Standard Deviation)—is the theoretical or empirical normal oscillation error around the central tendency: the inverse of N as the arithmetic mean of the empirical probabilities and the theoretical probability of occurrence by chance under equal opportunities.

5.5.5. Conclusion on the relationship between the inverse of N and the Mean or Standard Deviation in universes of infinite subjects or options

5.5.5.1. The inverse of N is the probability of theoretical dispersion or deviation—that is, the probability of theoretical bias. The Mean or Standard Deviation is the probability of sample empirical deviation or dispersion—that is, the probability of sample empirical bias.

5.5.5.2. As N increases, under normal conditions, the inverse of N and the Mean or Standard Deviation tend proportionally toward zero. The relationship between the increase in N and the tendency of the inverse of N and the Mean or Standard Deviation to zero is an inverse relationship of N to both the inverse of N and the Mean or Standard Deviation. The proportional trend toward zero of the inverse of N and of the Mean or Standard Deviation as N increases is a trend to zero of the inverse of N proportional to the trend to zero of the Mean or Standard Deviation, and vice versa, provided normal conditions hold.

5.5.5.3. When N decreases, under normal conditions, the inverse of N and the Mean or Standard Deviation increase proportionally. The increase of the inverse of N and of the Mean or Standard Deviation as N decreases is an inverse relationship of both the inverse of N and the Mean or Standard Deviation to N. As N decreases, the inverse of N increases, and the Mean or Standard Deviation increases proportionally, provided normal conditions hold.

5.5.5.4. “Normal conditions” obtain when the sample distribution conforms to what is normally regarded as normal: the random tendency of the sample to distribute around the arithmetic mean, with an error of plus or minus one in the distribution of standard scores. In a bias study where the positive or negative bias of some subject or option is increased, normal conditions do not necessarily hold; thus the Mean or Standard Deviation will depend on the experimental manipulation. In any case, a Validity of Bias—Positive or Negative—will only be universally valid after rational critique at the sample level accepts it, within the margin of moral responsibility—the error one is willing to accept. The same holds for equality of opportunity: a Validity of Equality will be universal when it is accepted in rational critique at the sample level.

5.5.5.4.1. In any case—and only in universes of infinite subjects or options—since the Maximum Theoretical Mean Deviation Possible is the average of twice the Maximum Theoretical Bias Possible, with the Maximum Theoretical Bias Possible equal to one minus the inverse of N, as the inverse of N tends to zero the Maximum Theoretical Bias Possible tends to one, and its double tends to two. Thus, logically, in an infinite universe, in bias studies the Maximum Theoretical Mean Deviation Possible tends to two over N, “2/N,” and as N tends to infinity the “2/N” term also tends to zero. Therefore, for both bias studies and equality-of-opportunity studies, as N tends to infinity the Maximum Theoretical Mean Deviation Possible or the Maximum Theoretical Standard Deviation Possible likewise tend to zero. That is, in all normal studies—both bias and equality—understood as those whose dispersion ranges between zero and a maximum, and only in universes of infinite subjects or options, as N increases the empirical and theoretical bias decreases inversely: the larger N is, the stronger the trend to zero of the inverse of N and the stronger the trend to zero of the Maximum Theoretical Mean or Standard Deviation; and for any normal study, the smaller N is, the greater the probability of theoretical bias (inverse of N) and the greater the probability of empirical bias (Mean or Standard Deviation). In Impossible Probability, “normal studies” are those where dispersion ranges between zero and a maximum, including bias or equality studies, to distinguish them from omega studies, where the ideals range between two and N minus one. We say normal conditions hold when, barring variations attributable to a variable, the model’s normal tendency would be to gravitate by chance toward the statistical

norm, the arithmetic mean, which facilitates the study of equality or bias because the conditions are ideal for attributing the detected equality or bias in the sample to the study variable; absent an affecting variable, the normal tendency by chance is to tend toward the mean.

5.5.5.5. Saying that normal error is plus or minus one on the normal curve implies that sample dispersion is error around the central tendency. The inverse of N, as the probability of sampling representativeness error and the probability of theoretical bias, is the theoretical normal oscillation error around the central tendency; and the Mean or Standard Deviation is the empirical normal oscillation error around the central tendency. The smaller the dispersion, the greater the representativeness of the central tendency, as both theoretical and empirical normal oscillation errors around the central tendency (the inverse of N) are reduced.

5.5.6. Conclusions on the relationship between the sample and the probability of empirical or theoretical dispersion/deviation

5.5.6.1. In universes of infinite subjects or options

5.5.6.1.1. The inverse of N is the arithmetic mean of the empirical probabilities, in any kind of universe.

1/N = arithmetic mean of the empirical probabilities in any kind of universe

5.5.6.1.2. The inverse of N is equal to the theoretical probability by chance—of equal opportunity; that is, it equals the theoretical behavior under equal conditions for every subject or option: the theoretical probability of chance, in any kind of universe.

1/N = theoretical probability of chance (chance-as-theory) in any kind of universe

5.5.6.1.3. The inverse of N equals the probability of sampling representativeness error in universes of infinite subjects or options, since the larger N is, the lower the probability of sampling representativeness error, and the smaller N is, the higher that probability; that is, the representativeness error is inverse to N—hence the inverse of N is the probability of sampling representativeness error.

1/N = probability of sampling representativeness error in universes of infinite subjects or options

5.5.6.1.4. The inverse of N, as the theoretical probability of dispersion or deviation, is the theoretical probability of bias; under normal conditions it is directly proportional to the probability of the Mean or Standard Deviation, while the Mean or Standard Deviation is the probability of sample empirical deviation or dispersion—that is, the probability of the sample's empirical bias. The Average of the Levels of Bias of the empirical probabilities can only be the central tendency of the Levels of Bias; any average of a probability differential is the sample probability of that probability differential.

1/N = theoretical probability of dispersion or bias in universes of infinite subjects or options

DMoS = probability of empirical dispersion or bias

5.5.6.1.5. The relationship between the probability of empirical or theoretical dispersion or deviation—that is, the relationship between empirical and theoretical bias, i.e., between the inverse of N and the Mean or Standard Deviation ("DMoS")—is directly proportional.

5.5.6.1.6. The relationship between N and the probability of empirical or theoretical dispersion or deviation—that is, the relationship between N and the probability of empirical or theoretical bias—is inversely proportional: the larger N is, the smaller the Mean or Standard Deviation and the smaller the inverse of N; the larger N is, the lower the probability of empirical or theoretical bias. Thus, the smaller N is, the higher the probability of empirical or theoretical bias; the smaller N is, the larger the Mean or Standard Deviation and the inverse of N.

5.5.6.1.7. The relationship between the probability of sampling representativeness error and the probability of dispersion or deviation—empirical or theoretical—is directly proportional. If the relationship between the probability of theoretical bias, 1/N, is directly proportional to the Mean or Standard Deviation, and the probability of theoretical sampling representativeness error equals the inverse of N, then the probability of sampling representativeness error is necessarily directly proportional to the Mean or Standard Deviation.

5.5.6.1.8. In universes of infinite subjects or options, the inverse of N is at once the theoretical probability of chance, the theoretical probability of sample bias, and the probability of sampling representativeness error, being directly proportional to the Mean or Standard Deviation, which is the probability of sample empirical bias (typical sample empirical bias). Bias and chance are identical within the margin of error—or moral responsibility—one is willing to accept: namely, the inverse of N, in universes of infinite subjects or options. Empirical bias and theoretical chance are identical because they are proportional in their relationship inversely proportional to N. Theoretical bias and theoretical chance are identical because they are proportional in their relationship inversely proportional to N. Empirical bias, theoretical bias, and theoretical chance are identical within the accepted margin of error—or moral responsibility—the inverse of N as the inverse of the sample, in universes of infinite subjects or options. The first decision of scientific policy must be to accept a margin of error; the first margin of error accepted by scientific policy is the margin of error derived from accepting the sample itself—accepting a sample is, in itself, accepting an error. The relationships of chance and empirical or theoretical bias in the sample will be relationships within the margin of error that is the sample itself. ***To accept a sample is***

to accept a moral responsibility toward the sample. Accepting a magnitude N implies accepting a margin of sampling representativeness error—the error of whether the sample is or is not representative—and implies accepting a theoretical error of normal oscillation around the central tendency. ***Chance, bias, and error*** are, theoretically, three different yet dialectically identical logical moments of the same process—the same sample—identified in the inverse of the sample itself. To accept a sample is to accept, politically, the chance, bias, and error of that sample. Even if, in a given empirical model, scientific policy were to accept only a critical probability—zero percent error in an error study, or one hundred percent reliability in a reliability study—in every case, always and without exception, the moral judgment in the statistical decision will be directly proportional to the margin of error accepted in the sampling itself. Every sample accepted by scientific policy implies that scientific policy accepts the risks—errors—morally derived from that sample: the inverse of the sample, the inverse of N in universes of infinite subjects or options, and the inverse of the direct scores or frequencies in universes of limited options. Even if scientific policy decides to include an entire population, consisting of a population of N subjects or options, in the study, that population N itself becomes a sample for the study; and the population, as a sample, will have a probability of sampling representativeness error equal to the inverse of N, and a probability of theoretical normal oscillation error around the central tendency equal to the inverse of N, with the inverse of N being equal to that same central tendency and theoretical dispersion of the sample. Every population consisting of N subjects or options, in which a particular quality is studied, must be included among universes of infinite subjects or options, even if it appears not to be infinite. In the infinite, nothing is as it seems.

5.5.6.1.9. If the Mean Deviation is the distribution of Total Bias by chance, then the Mean Deviation is the sample of chance—the empirical chance of the sample—directly proportional to the inverse of N, theoretical chance. Empirical chance and theoretical chance are directly proportional to the same extent that empirical bias and theoretical bias are directly proportional. The empirical and the theoretical are identical within a possible margin of doubt, insofar as every objective idea morally represents a real identity between itself and what it mentally represents, within a margin of uncertainty

in which that identity may be false. The material structure in which the idea is represented is the brain; every idea is a model of the brain. Depending on the type of study—bias or equality of opportunity; error or reliability—the structure of the margin of error or of reliability will accord with the object of study; in any case, in rational critique on critical probability the margin of error or reliability will be determined by the percentage X of error or reliability that scientific policy accepts.

5.5.6.2. In universes of limited options

5.5.6.2.1. In universes of limited options, the inverse of N, 1/N, is the arithmetic mean and the theoretical probability of equal opportunity of occurrence by chance. The inverse of N may have other functions, but specifically, in two-option models it gives the greatest empirical dispersion and the probability of representativeness error for discrete categories.

5.5.6.2.2. In universes of limited options—for example, heads or tails when tossing a coin, or one or another candidate winning an election—the probability of sampling representativeness error and the probability of theoretical dispersion are theoretical probabilities of error and dispersion directly proportional to the inverse of the direct scores or frequencies, $1/\sum x_i$.

$1/\sum x_i$ = probability of theoretical error and dispersion in universes of limited options

5.5.6.2.3. In elections, the results will be more representative the more citizens exercise their right to vote.

5.5.6.2.4. In a study of whether a coin is biased, the more tosses are made, the more representative the study will be.

5.5.6.2.5. The number of voters for each candidate in an election is that candidate's vote frequency; the number of votes for a candidate is the number of times a vote in favor of that candidate is repeated; the number of votes for each candidate is the candidate's direct score in the election.

5.5.6.2.6. The number of times heads or tails comes up when tossing a coin is the frequency of heads or tails; the frequency of heads is the direct score for heads, and the frequency of tails is the direct score for tails.

5.5.6.2.7. Direct score and frequency are identical because they are opposites; every frequency is a direct score, and every direct score is a frequency—within the accepted margin of error, namely the inverse of the direct scores or frequencies.

5.5.6.2.8. Under normal conditions, if the coin is not biased, as the number of tosses increases the normal tendency is for the distribution of heads and tails to become normal; the probability of heads equals the inverse of N, and the probability of tails equals the inverse of N. Under equal opportunities in the absence of bias, the normal tendency is for every option to approach the inverse of N—equality of opportunities.

5.5.6.2.9. In democratic elections, being a bias study in which what is measured is the positive bias of the most-voted candidate, equality of opportunity does not hold: the object of study is the positive bias of the most-voted candidate. What is studied is the candidates' bias—or, in any case, that a significant majority of voters under equal conditions vote for the same candidate, giving that candidate a positive bias over the others. In any case, the larger the number of voters, the more reliable it is that the positive bias of the most-voted candidate is a representative bias of the population's political ideas. In a bias study, the greater the sample's empirical dispersion—Mean or Standard Deviation—the greater the significance of individual biases. Validity of Bias—Positive or Negative—will be universal when sufficiency is demonstrated at the sample level.

5.5.6.2.10. In conditions of equal opportunity, in a study of equal opportunity, the greater the number of random coin tosses—that is, the larger the sum of direct scores or frequencies, $\sum x_i$—the lower the probability of theoretical dispersion, the lower the theoretical probability of bias; that is, theoretical dispersion (theoretical bias) is the inverse of the direct scores or frequencies, just as, the greater the number of tosses, the lower the probability of sampling representativeness error when tossing a coin at random.

1/∑xi = probability of theoretical dispersion or bias in universes of limited options

5.5.6.2.11. In bias studies, the greater the increase in direct scores or frequencies, ∑xi, the lower the error of representativeness of the sample results (1/∑xi); the greater the number of voters, the lower the representativeness error of the election.

1/∑xi = probability of sampling representativeness error in universes of limited options

5.5.6.2.12. Focusing on a study of equal opportunity: under normal conditions, if increasing the number of direct scores or frequencies, ∑xi, lowers the probability of theoretical error and dispersion (lowers the theoretical probability of bias), then, likewise, as the number of direct scores or frequencies increases, the Mean or Standard Deviation—the empirical chance or empirical bias of the sample—also decreases. In every type of study, in addition to individual Validity, rational critique at the sample level is absolutely necessary.

5.5.6.2.13. Therefore, under normal conditions, if increasing the sample of direct scores or frequencies, ∑xi, decreases the probability of theoretical error or dispersion—decreases the theoretical probability of bias, 1/∑xi—then, proportionally, the Mean or Standard Deviation—the probability of empirical bias or empirical chance—also decreases. Hence the sample of direct scores or frequencies, ∑xi, is inversely proportional to the probability of theoretical error or bias, 1/∑xi, and the sample of direct scores or frequencies is inversely proportional to the probability of empirical bias or empirical chance, the Mean or Standard Deviation. Rational critique at the sample level, in equality of opportunity, must validate that Validity of Equality is truly universal.

5.5.6.2.14. The relationship between the sample of direct scores or frequencies, ∑xi, and the probability of theoretical bias—i.e., theoretical deviation or dispersion—is an inverse proportional relationship. Therefore, the inverse of the sample of direct scores

or frequencies equals the probability of theoretical deviation; that is, the inverse of the sample's direct scores or frequencies equals the probability of theoretical bias. This inverse relationship amounts to dividing the sample of direct scores or frequencies into one: 1/∑xi. The quotient of unity by the sample's direct scores or frequencies equals the probability of theoretical bias, and likewise unity over the sample of direct scores or frequencies equals the probability of sampling representativeness error in universes of limited options.

5.5.6.2.15. Any change in the sample of direct scores or frequencies, ∑xi, under normal conditions, will imply inversely proportional changes in the probability of theoretical error and dispersion or deviation—the theoretical probability of error and bias, 1/∑xi—while those changes in theoretical bias and error will be directly proportional to the changes in the Mean or Standard Deviation, under normal conditions. The larger the sample of direct scores or frequencies, the lower the probability of theoretical error and bias and the lower the Mean or Standard Deviation. The smaller the sample of direct scores or frequencies, the higher the probability of theoretical error and bias, and the higher the Mean or Standard Deviation, in universes of limited options.

5.5.6.2.16. Thus, under normal conditions, any change in the inverse of the direct scores or frequencies, 1/∑xi—the probability of theoretical error and bias in universes of limited options, i.e., the probability of error and the probability of theoretical deviation or dispersion—will be changed directly proportional to the probability of empirical deviation or dispersion, the Mean or Standard Deviation.

5.5.6.2.17. Within the possible universes of limited options, a special case is the universe limited to two options. In two-option universes the inverse of N performs one more function that it does not perform in any other model. In two-option models the inverse of N, equal to the inverse of duality, 1/2, is the greatest Mean or Standard Deviation that can exist in two-option models.

5.5.6.2.18. In two-option models, as in any other universe of limited options, the inverse of the direct scores or frequencies is the probability of theoretical bias—i.e., theoretical dispersion or deviation—just as the Mean or Standard Deviation is the probability of empirical bias. The only particularity in two-option universes is that, under certain

circumstances of increasing bias, the Mean or Standard Deviation may be equal to or greater than the inverse of the direct scores or frequencies, but absolutely never— in two-option models—will the Mean or Standard Deviation be greater than the inverse of N; at most, Mean or Standard Deviation will equal the inverse of duality, the inverse of N in two-option models. But in two-option models, where N equals two, the inverse of N— the inverse of duality—is 0.5. In two-option models, never, absolutely never, can the Mean or Standard Deviation be greater than the inverse of N; at most it can equal the inverse of N under conditions of maximum possible empirical dispersion, and only in the two-option model.

5.5.6.2.19. In universes of limited options, insofar as the inverse of the direct scores or frequencies is the probability of sampling representativeness error and the probability of theoretical dispersion, the synthesis of both functions means that, in universes of limited options, the inverse of the direct scores or frequencies will be the theoretical error of normal oscillation around the central tendency— the central tendency always being, in any kind of universe, the inverse of N—while the Mean or Standard Deviation is the empirical error of normal oscillation around the central tendency, the inverse of N. Since, with a larger sample of direct scores or frequencies, the central tendency is more representative, the relationship between the theoretical oscillation error around the central tendency and the sample of direct scores or frequencies must be an inverse proportional relationship, and likewise the relationship between the sample of direct scores or frequencies and the empirical error of normal oscillation around the central tendency must also be inversely proportional; empirically, the normal oscillation around the inverse of N is the Mean or Standard Deviation in universes of limited options. In a bias study, the opposite of what normally occurs may happen: if normally, as the sample increases, dispersion tends to decrease, in a bias study where biases are pushed to their maximum possible values, it may happen that increasing the sample of direct scores or frequencies does not necessarily imply a reduction in dispersion. Even so, it is true that as the sample of direct scores or frequencies increases, it is always more likely that dispersion will tend to moderate, even in a bias study. Normally, in democratic elections, the greater the electoral participation—the more people exercise their right to vote—the more the vote differences between the main candidates tend to

narrow. In general, in a universe of direct scores or frequencies, even in a bias study, the possibility of including more direct scores or frequencies in the model will normally tend to reduce empirical dispersion and, logically and necessarily, theoretical dispersion as well. Of course, among infinitely many direct scores or frequencies, nothing is as it seems: it may occur that in a democracy absolutely all citizens vote for the same candidate because that candidate alone inspires confidence; or the opposite—amid **political uncertainty** no candidate receives positive votes; or that, at a certain point in the universe under certain physical circumstances, absolutely all **photons behave as particles, or only as waves**. Normally, the more direct scores or frequencies increase in a universe of limited options—even in a bias study—the differences between the different options will tend to moderate; but this need not happen absolutely: there may be limited-option models in which, **under certain circumstances, frequency behaves in an all-or-nothing fashion**—under some circumstances one option may have zero frequencies and the opposite option receive all the frequencies, and vice versa under opposite circumstances.

5.6. Study and acceptance of sampling representativeness error and theoretical dispersion

5.6.1. Introduction

5.6.1.1. Every statistical decision implies accepting errors. The statistical errors that scientific policy must be willing to accept are the sampling representativeness error and, for inferential studies, in addition, the margin of error of critical reason itself.

5.6.1.2. Every statistical decision will always have an associated margin of error at least equal to the sampling representativeness error; and, moreover, in those inferential studies in which acceptance of the empirical hypothesis depends on a critical reason that accepts a margin of logical or moral incoherence (error), the margin of error will, in any case, be double: in addition to the sampling representativeness error—the inverse of the sample—there is the margin of error of critical reason itself.

5.6.1.3. In universes of infinite subjects or options, the inverse of N is at once the probability of sampling representativeness error and the probability of theoretical

dispersion, as well as the arithmetic mean of the empirical probabilities and the probability of equality of opportunity by chance.

5.6.1.4. Whereas in universes of limited options, the inverse of the direct scores or frequencies will be the probability of sampling representativeness error and the probability of theoretical dispersion, while the inverse of N will be the arithmetic mean and the probability of equality of opportunity by chance.

5.6.1.5. Insofar as the inverse of the sample— the inverse of N in universes of infinite subjects or options, and the inverse of the direct scores or frequencies in universes of limited options— is at once theoretical error and theoretical bias in their respective universes, one way to control theoretical error and bias in the sample is through the rational critique of the magnitude of the sample itself.

5.6.1.6. If the inverse of the sample is directly proportional to the margin of empirical error and dispersion of the sample—i.e., the inverse of the sample is directly proportional to the Mean or Standard Deviation under normal conditions—one logical way to study whether normal conditions hold is through a rational critique of the sample itself.

5.6.1.7. The rational critique of the sample can be done in two different ways: either through Sample Sufficiency, in any kind of universe and form of direct score or frequency; or, albeit more restrictively, through the Normality Requirement or the Critical Normality Requirement, valid for any universe of limited options and, in universes of infinite subjects or options, depending on whether the direct score measurement contains a value equal to or greater than one; and, depending on the object of study, the Normality Requirement in studies of moderation or equality of opportunity, and the Critical Normality Requirement in studies of positive bias and in those studies of equality of opportunity in which the direct score of each subject or option must be greater than one.

5.6.1.8. Sample Sufficiency will be the rational critique directly of the inverse of the sample relative to a critical probability: if the inverse of the sample—whether the inverse of N in universes of infinite subjects or options, or the inverse of the direct scores or frequencies in universes of limited options—is equal to or less than the

critical probability, then the sample is accepted, since the probability of sampling representativeness error lies within the margin of error accepted by the scientific policy's critical reason. The only limit to Sample Sufficiency for the sample of direct scores or frequencies is that it cannot be used in studies where the ideal of the study is the zero sample.

5.6.1.9. The Critical Normality Requirement will be valid for any study in a universe of limited options where the object of study is positive bias, and in those equality-of-opportunity studies where the frequency must likewise be greater than one; and in universes of infinite subjects or options—whether a study of positive bias or of equality of opportunity—where the direct score of each subject or option is greater than one. The Normality Requirement applies to any study of moderation or equality of opportunity in universes of limited options, and in universes of infinite subjects or options in any study of moderation or equality of opportunity provided that the direct score for each subject or option is at least equal to one.

5.6.1.10. The zero sample (*muestra de ceros*) is a type of sample characterized by the study's ideal being that all empirical probabilities of all subjects or options, equally, be equal to probability zero. Evidently, if the object of study is that the direct scores or frequencies be probability zero, this is the only study model in which, if it occurs—especially in a universe of limited options—the inverse of the direct scores or frequencies will not itself be the probability of representativeness error of the sample of direct scores or frequencies, since if the ideal is the reduction of all direct scores or frequencies to zero, then the direct scores or frequencies, as the object to be reduced, are themselves a non-ideal factor; therefore the sample of direct scores or frequencies would itself be the factor of error.

5.6.1.11. Evidently, in those studies where the ideal of the sample is the zero sample, Sample Sufficiency of the sample of direct scores or frequencies will make no sense.

5.6.1.12. In the case of zero-sample studies: only if, over more than one measurement—for example, in two measurements—the ideal were that at least the first measurement be as representative as possible, in order to study the trend toward zero in subsequent measurements starting from a first sufficiently representative

measurement, then for that first measurement only the Sample Sufficiency study of the sample of direct scores or frequencies would be a valid model.

5.6.1.13. In every model of rational critique in *Probabilidad Imposible*, the way to represent the critical probability mathematically will always be by the symbol "p(xc)," and it will be calculated as the product of the percentage X (of error or reliability) over one hundred by a maximum or ideal trend.

p(xc) = critical probability

5.6.1.14. In Sample Sufficiency, insofar as the inverse of the sample is a probability of error, the maximum representativeness error would occur if the inverse of the sample—whether N or direct scores or frequencies, depending on the universe—were equal to one. If the maximum error in the inverse of the sample is one, then in Sample Sufficiency the critical probability will be the percentage X of error over one hundred which, multiplied by the maximum error (unity), equals that same quotient of percentage X of error over one hundred. Since the maximum possible error is the absence of indeterminism, the maximum possible error in models of infinite subjects or options is that there is only one subject or option (N = 1), or in models of limited options that the sample has only one occurrence (xi = 1)—models of maximum error because they would not be probability studies, for there to truly be probability it is necessary to have more than one subject or option and more than one occurrence, unless the ideal is the zero sample where there may be zero occurrences, provided that that is the ideal. In any case, if a sample consists of a single subject or option, it would not be a stochastic model; it would be deterministic. ***The maximum error is to believe a deterministic model is indeterministic,*** [or the opposite, it is a mistake to believe that something indeterministic is deterministic, to be wide open *means* cognitive flexibility].

5.6.1.15. In the Critical Normality Requirement, in studies of positive bias and certain studies of equality of opportunity, insofar as the ideal is that the inverse of the direct scores or frequencies be equal to or less than, by a critical value, the inverse of N, the

critical probability will be equal to the inverse of N times the reliability percentage X over one hundred, since the inverse of N is the maximum value toward which the difference of the inverses of the samples can tend.

5.6.1.16. In the Normality Requirement, in a study of moderation or equality of opportunity, one directly tests whether the inverse of the direct scores or frequencies is equal to or less than the inverse of N.

5.6.1.17. In Probabilidad Imposible the order of the factors in every model of differential rational critique must be such that only zero or positive is accepted; since the spring of 2001 the aim of Probabilidad Imposible has been to transform the number zero into a universal critical value.

5.6.1.18. For anything to be rational it suffices that it be sufficient: in an error study, a sufficient trend is one that is equal to or less than the critical threshold as a margin of error; in a reliability study, a trend is sufficient if it is equal to or greater than the critical threshold as a margin of reliability. In the Normality Requirement it is enough that the inverse of the direct scores or frequencies be equal to or less than the inverse of N to be sufficient, insofar as the inverse of N itself, in the Normality Requirement, acts directly as the margin of error.

5.6.1.19. In a critical study, if the critical probability is calculated on the percentage X of error, it is a margin of error; if it is calculated on a percentage X of reliability, it will be a margin of reliability.

5.6.2. Sample Sufficiency

5.6.2.1. Sample Sufficiency: a sample is sufficient if the inverse of the sample is equal to or less than a critical probability—i.e., a margin of sampling representativeness error one is willing to accept—except, in universes of limited options, for those samples of direct scores or frequencies whose ideal is the zero sample, because in that case the error is in fact the sample of direct scores or frequencies itself.

5.6.2.2. Sample Sufficiency in universes of infinite subjects or options: critical probability minus the inverse of N—if zero or positive, the sample N is accepted as

sufficiently representative. The critical probability equals the percentage X of error over one hundred.

Sample Sufficiency = p(xc) – 1/N = zero or positive → the sample is accepted

p(xc) = X : 100

X = percentage of error

5.6.2.3. Sample Sufficiency of the sample of direct scores or frequencies, except for those studies whose ideal is the zero sample:

Sample Sufficiency = p(xc) – 1/∑xi = zero or positive → the sample is accepted

p(xc) = X : 100

X = percentage of error

5.6.3. Critical Normality Requirement and Normality Requirement

5.6.3.1. The Normality Requirement and the Critical Normality Requirement will be valid in any universe of limited options; and, in universes of infinite subjects or options, the Normality Requirement applies so long as at least all direct scores or frequencies are equal to or greater than one, and the Critical Normality Requirement applies when they are greater than one. The Critical Normality Requirement will be applied to studies of positive bias and to those studies of equality of opportunity where each subject or option alike must have a direct score or frequency greater than one, in any kind of universe. The Normality Requirement will be applied to studies whose object is the moderation of direct scores or frequencies, or equality of opportunity, whether in

universes of limited options or of infinite subjects or options—although in the latter at least the direct scores must be equal to or greater than one.

5.6.3.2. Insofar as the Critical Normality Requirement or the Normality Requirement demand a particular mode of direct score or frequency equal to or greater than one, zero samples are excluded from these tests; moreover, in universes of infinite subjects or options, measurements where the direct scores are decimal values without an integer part would also be excluded.

5.6.3.3. In summary, the Critical Normality Requirement and the Normality Requirement will only be valid in those universes where the sample—whether N in universes of infinite subjects or options, or direct scores or frequencies in universes of limited options—is required to be sufficiently representative to be normal; in universes of infinite subjects or options, the measurements must include an integer part, or in universes of limited options the sum of the frequencies must be at least equal to N.

5.6.3.4. Thus, whenever the preliminary conditions of the Normality Requirement or the Critical Normality Requirement are met, they will be applied, if scientific policy so deems, to the rational critique of the sample; and if the preliminary conditions are not met, only Sample Sufficiency may be applied. In any case, Sample Sufficiency would already be, in itself, a sufficient rational test without the need for the Normality Requirement or the Critical Normality Requirement, even if the preliminary conditions were satisfied.

5.6.3.5. In those studies of equality of opportunity and positive bias that so require, both for universes of infinite subjects or options and of limited options, the Critical Normality Requirement equals the inverse of N minus the inverse of the direct scores or frequencies, and the result minus a critical probability; if the result is zero or positive, normality is accepted—whether the sample is N in universes of infinite subjects or options, or the sample of direct scores or frequencies in universes of limited options. Critical probability equals the product of the inverse of N by the reliability percentage X over one hundred.

Critical Normality Requirement =

(1/N – 1/∑xi) – p(xc) = zero or positive → normality accepted

p(xc) = 1/N · (X : 100)

X = percentage of reliability

5.6.3.6. The Normality Requirement, in studies of equality of opportunity or where the aim is simply to moderate the direct scores or frequencies for every subject or option alike, without reducing them to zero, equals the difference of the inverse of N minus the inverse of the direct scores or frequencies; zero or positive is accepted.

Normality Requirement =

1/N – 1/∑xi = zero or positive → normality accepted

5.6.3.7. Depending on the study model, scientific policy must decide whether to apply Sample Sufficiency, the Critical Normality Requirement, or the Normality Requirement; in any case, whether or not the preliminary conditions are met, Sample Sufficiency would already be, by itself, a sufficient rational test for validating the sample.

5.6.4. Synthesis of the rational critique of the sample

5.6.4.1. There are fundamentally two ways to critique the sample for acceptance: Sample Sufficiency and the Critical Normality Requirement or Normality Requirement.

5.6.4.2. Sample Sufficiency critiques the inverse of the sample, examining whether the inverse of the sample is equal to or less than a critical threshold (percentage X of error over one hundred), and may critique the inverse of N in universes of infinite subjects or options, or the inverse of the direct scores or frequencies in universes of limited options.

5.6.4.3. The Critical Normality Requirement studies the relationship between both sample models for every possible universe, but only for studies of positive bias and certain studies of equality of opportunity, and always provided the preliminary conditions are met, such that the difference between the inverse of N and the inverse of the direct scores or frequencies must be equal to or greater than a critical threshold, which equals the reliability percentage X over one hundred times the inverse of N.

5.6.4.4. The Normality Requirement likewise studies the relationship between both samples, but for any study of equality or moderation—the object of moderation is explained in Section Ten—such that if the inverse of the direct scores or frequencies is equal to or less than the inverse of N, normality is accepted.

5.6.4.5. The importance of all these models for accepting the sample, whether according to the type of universe (Sample Sufficiency) or according to the object of study (Critical Normality Requirement or Normality Requirement), is that they allow objective studies of whether a sample is accepted as sufficiently representative.

5.6.4.6. Insofar as the probability of sampling representativeness error and the bias of the sample are directly proportional to the inverse of the sample, one way to study whether to accept the representativeness error and the bias of the sample is through the rational critique of the sample itself, in order to determine—on critical values—the magnitude of error or bias that scientific policy is willing to accept in the study.

5.7. Normal Level of Bias

5.7.1. The empirical probability is the empirical trend of each subject or option.

5.7.2. The theoretical probability equals the behavior expected by chance for every subject or option under equality of opportunity; the theoretical probability, the inverse of N, 1/N, is—within the Normal Level of Bias—the theoretical probability of chance.

5.7.3. The difference between empirical probability and theoretical probability is the Normal Level of Bias; if it is negative it is called negative bias, and if it is positive it is called positive bias.

5.7.4. What in *Probabilidad Imposible* is known as the Normal Level of Bias is what in conventional or classical statistics—the First Method—is known as the differential score. In *Probabilidad Imposible* the Normal Level of Bias differs from other possible models of relative Levels of Bias in that the Normal Level of Bias is calculated on the difference between empirical probability and theoretical probability, whereas the relative Levels of Bias—explained in Section 14—are estimated on the difference between each empirical probability and any other individual statistic except the inverse of N. Since, most commonly, "Level of Bias" refers to the difference between empirical probability and the inverse of N, whenever "Level of Bias" is mentioned in *Probabilidad Imposible* without any other qualifier (normal or relative), it is understood to refer to the Normal Level of Bias—empirical probability minus theoretical probability

Normal Level of Bias or Level of Bias = $p(xi) - 1/N$

5.7.5. The average of the absolute values of the Level of Bias is the Mean Deviation, which represents the distribution of Total Bias by chance—Total Bias times the inverse of N; the sum of the absolute values of the Levels of Bias times the inverse of N. Bias-by-chance or chance-by-bias: the order of the factors does not change the product. The Mean Deviation shows the empirical behavior of chance in the sample. The Mean Deviation is at the same time the sample's empirical bias and the sample's empirical chance—an empirical error of normal oscillation around the central tendency.

5.7.6. For the summation of the Levels of Bias to be possible, the differentials must be taken in absolute terms. Owing to the natural goodness of chance, the only way to compute the sum of Levels of Bias—so as to neutralize the dialectical identity whereby the positive tends to offset the negative—is by summing the absolute values of the normal Levels of Bias. Natural goodness—the goodness of chance—in the inverse of N,

as the midpoint of the sample (the arithmetic mean of the empirical probabilities), will make the sum of all positive normal Levels of Bias equal to the sum of all negative normal Levels of Bias. This is an essential property of the natural goodness of the inverse of N, which will split the Total Bias so that Total Bias divided by two equals the sum of all positive Levels of Bias, and likewise Total Bias divided by two equals the sum of all negative Levels of Bias. This equation—Total Bias divided by two—will be called the Maximum Possible Empirical Bias, and it will be an essential formula in some models of rational critique. One convention in *Introduction to Impossible Probability* is that whenever a summation sign appears before a bracket, it denotes the summation of all the sample operations inside that bracket; and when, at the end of the bracketed summation, a division appears, what is divided is the entire bracketed summation by that divisor. In this case, the sum of the absolute values of the Levels of Bias—Total Bias—divided by two: Total Bias over two equals Maximum Possible Empirical Bias.

Maximum Possible Empirical Bias = $\sum | p(xi) - 1/N | : 2$

$\sum | p(xi) - 1/N |$ = sum of all unsigned Levels of Bias, i.e., Total Bias

$\sum | p(xi) - 1/N | : 2$ = Total Bias over two, Maximum Possible Empirical Bias

5.7.7. The average of the Levels of Bias—the Mean Deviation, the Average Level of Bias—is an absolute (unsigned) value, so it does not measure whether there is more positive or more negative bias, because there is as much positive bias offset by negative bias as there is negative bias offset by positive bias, and the sum of all positives and negatives, by logical consequence of natural goodness, would be zero if the sign were taken into account. The Maximum Possible Empirical Bias—Total Bias over two—measures what can be the maximum empirical bias of a single subject or option. In studies of positive bias, if the ideal were that only one subject or option has positive bias and all the rest negative bias, that sole subject's positive bias would equal the Maximum Possible Empirical Bias, i.e., Total Bias over two. If the aim is that all subjects

or options have positive bias except for a single subject or option that must have negative bias, then the negative bias of that unique subject or option should equal the Maximum Possible Empirical Bias, i.e., Total Bias over two.

5.7.8. The average amount of absolute bias—the Mean or Typical Deviation—is the average sample amount of bias that separates empirical behavior from theoretical behavior. If the Total Bias were distributed under equality of opportunity, what portion of bias should fall to each subject or option? Logically, half the sample should have empirical probability equal to the inverse of N plus the Mean or Typical Deviation, and the other half equal to the inverse of N minus the Mean or Typical Deviation, if the bias were distributed equally in the sample. If N is odd, one subject or option more would have positive or negative bias; therefore, the sum of that extra positive or negative bias divided by the other half of the sample should be subtracted from or added to the inverse of N to determine the negative or positive bias of the other half of the sample.

5.7.9. The average absolute amount of bias—the Mean or Typical Deviation—is the average sample amount of bias that separates empirical behavior from theoretical behavior.

5.7.10. In infinite universes, as the sample grows, empirical probabilities tend to zero; the Levels of Bias tend to zero; and as the Levels of Bias tend to zero, the difference between empirical behavior and behavior under equality of conditions, by chance—that is, theoretical behavior—tends to zero. As N increases in universes of subjects or options that are infinite, or as the sample of direct scores or frequencies "∑xi" increases in universes of limited options, under normal conditions of equality of opportunity, empirical behavior tends to match behavior by chance as the sample grows—whether the sample is N (in universes of infinite subjects or options) or ∑xi (in universes of limited options). In either kind of sample, in its respective universe, whenever the sample increases under normal conditions, the Mean or Typical Deviation will tend to behave directly proportionally to the inverse of the sample: in universes of infinite subjects or options, the inverse of N; in limited-option universes, the inverse of the direct scores or frequencies. As N increases in universes of infinite subjects or options—or as ∑xi increases in universes of limited options—every subject or option will tend toward the inverse of N, provided equality of opportunity holds. In the tendency to zero in truly

infinite universes—in the tendency to zero as N increases in universes of infinite subjects or options—the greatest equality of opportunity holds; in a truly infinite universe, in all its dimensions, empirical chance tends to identify with theoretical chance without error: in the infinite, everything is equally possible.

5.7.11. If, in a truly infinite universe in all its dimensions—a universe of infinite subjects or options in infinite time—absolutely every margin of logically coherent possibility, however seemingly impossible, is inevitable; nevertheless, at the level of moral existence, in a universe limited to existing moral options, only those options coherently limited by moral existence will be possible. In a universe limited to immoral options—where it is immoral to accept a demonstrably false proposition that an immoral policy refuses to recognize as false for its own interests—the immoral is inevitable. In a universe limited to both moral and immoral options, the moral option will be inevitable at least at some moment in spacetime, though if the moral option is simultaneous with the immoral option, mere simultaneity does not sufficiently guarantee truth, because just as the moral can be inevitable, so too can the immoral be inevitable. Only in a universe limited exclusively to moral options—where morality consists in rejecting the demonstrably false proposition in order to replace it with a more isomorphic proposition—***will scientific progress be accelerated***. Any proposition with a margin of error is a proposition that is coherently moral over a margin of logical or moral incoherence. Moral incoherence, being immoral, should be minimized as much as possible. What is coherently moral will be identical to moral incoherence—the immoral—within the margin of error. Any morality accepted with a margin of error—immoral—will, inevitably, when truth comes due, be a moral incoherence in truth. However, ***when that moment of truth arrives, true morality is that which rejects the refuted truth to replace it with a higher or deeper form of moral coherence. Scientific progress***, in the dialectic of the moral and the immoral, should be humanity's progress toward the formulation of provisional or historical universal maxims—relative universal maxims.

5.7.12. ***In an infinite universe in all its dimensions, what is scientific, logical, and coherent—possible or impossible—will be an inevitable truth***, *within the margin of empirical skepticism or rational doubt*. In a universe limited to what exists, only what

exists will be inevitable; the chief moral or political duty of critical reason is to defend the existence of life within chance, life in the free will of full, de facto, equality of opportunity.

5.8. The Level of Similarity

5.8.1. The Level of Similarity is one of the characteristic expressions of *Impossible Probability*.

5.8.2. Level of Bias and Level of Similarity

5.8.2.1. I formulated the idea of the Level of Similarity from my reflections on the Level of Bias—the differential score of empirical probabilities. I set out below my chain of reflections.

5.8.2.1.1. If A then B, it is that A causes B, within the logic of the syllogism of causality.

5.8.2.1.2. If A causes B, then as often as A occurs, so often will B occur.

5.8.2.1.3. If as often as A so often is B, then the frequencies of A and B tend to be the same—identical; that is, the difference between the frequency of A and the frequency of B should be zero.

5.8.2.1.4. The causal study between cause A and effect B is the study of the difference obtained by subtracting the frequency of B from the frequency of A.
5.8.2.1.4.1. The closer that A–B differential is to zero, the greater the correlation between A and B.
5.8.2.1.4.2. The more different from zero that difference is, the lower the correlation between A and B.

5.8.2.1.5. If A causes B, then the frequency of A will tend to be similar to that of B.

5.8.2.1.6. If the frequencies of A and B are similar, then the quotient of dividing the frequency of A by the frequency of B will be equal to one.

5.8.2.1.6.1. If the quotient of A and B is close to one, there is greater similarity and greater correlation. The principal studies in the First Phase of *Impossible Probability* in 2003 concerned creating new models of correlation between statistical probabilities.

5.8.2.1.6.2. If the quotient of A and B is far from one, there is less similarity and lower correlation.

5.8.2.1.7. In *Impossible Probability* the universal critical value is zero.

5.8.2.1.8. The base-10 logarithm of one equals zero, because the logarithm of ten is one.
5.8.2.1.8.1. If A and B are equal or nearly equal, then their quotient is equal to or tends to one, and its base-10 logarithm is zero or tends to zero.
5.8.2.1.8.2. If A and B are very different, then their quotient will be far from one and its base-10 logarithm far from zero.

5.8.2.1.9. The Level of Similarity of the empirical probabilities will equal the quotient of empirical probability over theoretical probability, with the result transformed via the base-10 logarithm; the closer the logarithm is to zero, the more similar; the further from zero, the less similar. It will be expressed as follows, where any logarithm written before a parenthesis denotes the logarithm of what is inside the parenthesis. In the first phase of *Impossible Probability* the theoretical probability of chance under equality of opportunity was denoted "p(xt)", and it is the inverse of N. Since the second phase I have preferred to write the inverse of N directly in the equations as "1/N." [Every time in GAI I mention the concept of similarity, especially regarding matching skills, my concept of similarity in matching skills is based on this equation]

$$\text{Level of Similarity} = \text{Log}_{10}\left(p(xi) : p(xt) \right) = \text{Log}_{10}\left(p(xi) : 1/N \right)$$

5.8.2.1.10. Interpretation of the Level of Similarity
5.8.2.1.10.1. Level of Similarity equal to zero ⇒ empirical probability and theoretical probability (the inverse of N) are identical.
5.8.2.1.10.2. Level of Similarity tending to zero with a negative sign ⇒ the empirical probability tends to be identical to the theoretical one and is less than the theoretical one.
5.8.2.1.10.3. Level of Similarity tending almost to zero with a positive sign ⇒ the

empirical probability tends to be identical to the theoretical one and is greater than the theoretical one.

5.8.3. The Level of Similarity when the empirical probability is zero

5.8.3.1. In base-10 logarithm theory, the base-10 logarithm of zero would be irrational, because the base-10 logarithm of zero does not exist.

5.8.3.2. In any case, the essential purpose of the Level of Similarity—as its name indicates—is the study of similarity rather than dispersion; therefore, its use is more suitable for measuring equality of opportunity.

5.8.3.3. However, within the Second Method there can be empirical probabilities equal to zero because that subject or option had a direct score or frequency of zero—hence empirical probability zero—in which case it would be irrational for it to have any similarity; or empirical probability zero because, despite having a direct score or frequency different from zero, the probability is a decimal sequence of zeros that ends in a nonzero value beyond the decimal cutoff, whether at a given, known term of the decimal sequence, or at a supposedly infinite (and thus unknown) decimal prolongation of the sequence.

5.8.3.4. For those empirical probabilities equal to zero because, within the decimal cutoff due to truncation, only a portion of the decimal sequence that was a string of zeros was selected, scientific policy may compute the Level of Similarity of the full decimal value if that nonzero value at which the decimal sequence ends is known.
5.8.3.5. If it is an infinite decimal sequence of zeros whose end is unknown, the only viable hypothesis is that the logarithm of the Level of Similarity is negative infinity.

6.Validity and Sample Level

6.1. Critical probability as a margin of critical bias

6.1.1. Critical probability is itself the ideal of critical reason—a rational value that acts critically as a criterion of rational sufficiency. In error studies it is normally computed as X percent error (X/100) times a maximum trend of error or bias; in reliability studies it is normally X percent reliability (X/100) times a maximal ideal trend of bias or reliability.

6.1.2. If, in Validity of Equality, critical probability minus the absolute value of a subject's/option's Normal Level of Bias is zero or positive, it is accepted; the ideal, under equality of opportunity, is that the absolute value of the Normal Level of Bias be equal to or less than the critical probability. In Validity of Positive Bias, Level of Bias minus critical probability, if zero or positive, is accepted; the ideal is Level of Bias equal to or greater than the critical probability. In Validity of Negative Bias, the differential of inverse of N minus empirical probability—then that result minus critical probability—if zero or positive, is accepted; the ideal is that the differential (inverse of N minus empirical probability) be equal to or greater than the critical probability. In each case, critical probability is computed on different maximum trends or ideals, and in each case it serves different functions. In Validity of Equality, critical probability is a margin of error; in Validity of Positive Bias or Negative Bias, it is a margin of reliability.

6.1.3. Validity of Equality is an error-study model: the critical probability is computed directly from an X percent error—the percentage of empirical skepticism, or moral responsibility, one is prepared to accept in the hypothesis. Validity of Positive Bias or Validity of Negative Bias are reliability-study models: the critical probability is computed from an X percent reliability; thus, in reliability studies the percent error equals one hundred minus the percent reliability, and that percent error is the percentage of empirical skepticism (or moral responsibility) within which scientific

policy renders a moral judgment of truth—the statistical decision.

6.1.4. Applying the critical-probability model to the Validity test—whether Validity of Equality, Validity of Positive Bias, or Validity of Negative Bias—is one way to apply critical reason to bias, among other possible models explained in *Introduction to Impossible Probability*. In any case, critical reason in *Impossible Probability* always follows the same pattern: critical examination of a trend to determine whether it is sufficiently rational, given the margin of empirical skepticism or rational doubt—moral responsibility, the margin of error—one is willing to accept in the causal explanation of truly real reality, morally material reality. One key feature of *Impossible Probability*, which will be clearly reflected when section eleven addresses various models of critical reason, is equifinality: the same object of study can be critically examined by different models of critical reason. Although only Validity and Sample Level are explained in this section, starting in section eleven, devoted to critical reason—critical probability—multiple ways of rationally critiquing bias or equality will be presented, both for individual dispersion and for sample dispersion. The existence of different ways to critique reality implies that the same reality in itself can be critiqued from different models of critical reason, which gives the study of trend greater possibilities for achieving the greatest possible isomorphism.

6.1.5. If, in Validity of Equality, the absolute value of a Level of Bias must be equal to or less than a critical probability, then this is an error-study model. Critical probability will be the margin of error one is prepared to accept in accepting the absolute value of a Level of Bias; it is computed as X percent error (X/100) times the maximum bias or error. Under equality of opportunity, the ideal is the absence of bias; therefore, any bias is error. The Maximum Theoretical Possible Bias will necessarily be the maximum individual error under equality of opportunity, of which, ideally, only the minimum possible error will be accepted: X percent error (X/100) times the Maximum Theoretical Possible Bias—i.e., the critical probability in Validity of Equality. The Maximum Theoretical Possible Bias equals the difference between the Maximum Possible Empirical Probability and the inverse of N. The Maximum Possible Empirical Probability

is, logically, one, because no empirical probability can exceed one; therefore, Maximum Theoretical Possible Bias equals one minus the inverse of N.

6.1.6. If the test, in a bias study where the ideal is to increase a certain bias, requires the normal bias to be equal to or greater than the critical probability, it is a reliability-study model, in which the critical probability is the margin of reliability one is prepared to accept in accepting the bias; it is computed as X percent reliability (X/100) times the maximum bias or maximum reliability. The ideal of bias will vary depending on whether it is a study of positive bias or of negative bias. In bias studies, logically, the greater the bias, the greater the reliability; thus bias is reliability, while chance is error, in studies of positive or negative bias.

6.1.7. Logically, in a study of positive bias, if in the entire N there is only a single ideal subject or option, the ideal must be the Maximum Theoretical Possible Bias—the difference between one and the inverse of N. Therefore, in Validity of Positive Bias, the Level of Bias must be equal to or greater than the critical probability, with the critical probability being the margin of reliability over the Maximum Theoretical Possible Bias: critical probability equals X percent reliability (X/100) times (1 – 1/N).

6.1.8. Logically, in a study of negative bias, the ideal negative bias will be the inverse of N itself, since the inverse of N is the Maximum Possible Negative Bias. Studies of negative bias examine the tendency of options toward zero. The Minimum Possible Empirical Probability is, logically, zero, because no empirical probability of a positive event can be less than zero. If the Level of Bias is the difference between empirical and theoretical probability, and negative bias occurs when empirical probability is less than theoretical probability, then the Maximum Possible Negative Bias can only occur when the empirical probability equals the Minimum Possible Empirical Probability—that is, empirical probability zero. Then, in the Level of Bias, zero minus the inverse of N equals negative inverse of N; the absolute value of negative inverse of N must equal the inverse of N. Therefore, in a study of negative bias the ideal absolute Level of Bias is to tend toward the Maximum Possible Negative Bias—the inverse of N. If, in a study of negative

bias, the differential (inverse of N minus empirical probability) is equal to or greater than the critical probability, negative bias is accepted as tending, sufficiently, to be a rational negative bias. The critical probability must equal X percent reliability (X/100) times the inverse of N, since within the multifunctionality of the inverse of N it now serves as the ideal of negative bias—the Maximum Possible Negative Bias.

6.2. Error and statistical inference

6.2.1. If scientific policy accepts a hypothesis because, in an error study, the empirical model is equal to or less than the ideal critical reason—critical probability—or because, in a reliability study, the empirical model is equal to or greater than the ideal critical reason—critical probability—then statistical inference for universal explanation or understanding (the *moral truth of the universe*, true reality in itself) will be provisional. That provisionality is directly proportional to the two accepted margins of error: sample selection and the percentage of error. The smaller the accepted error, the greater the likelihood the hypothesis will endure over time; the larger the margin of error, the less likely it will endure. In an error study, any X percent error used to calculate the margin of error is the X percent error that critical reason accepts in its rational critique. In a reliability study, any X percent reliability implies a percent error equal to one hundred minus the percent reliability; this percent error is the margin within which, in a reliability study, scientific policy renders a moral judgment of truth—the statistical decision. Any universal scientific inference will be provisionally true, with provisionality proportional to the accepted error: the error in accepting the sample (equal to the inverse of the sample) and the error percentage accepted by critical reason. The margin of error is itself a margin of empirical skepticism, insofar as accepting a margin of error in the causal explanation implies accepting a margin of causal indeterminacy; the margin of error will be a margin of rational doubt, because in any case it evidences a margin of incoherence in the scientific proposition; and the margin of error will be a margin of moral responsibility. If a pharmaceutical company develops a medicine to cure a disease, moral responsibility is directly proportional to the margin of error accepted in the experimental trials: ***the stricter the critical reason*** in accepting the smallest

possible error, the higher the moral standard of scientific policy; the less stringent the critical reason, accepting a lax margin of error, the lower the moral standard of scientific policy. Every margin of error entails a margin of moral incoherence in propositions; any moral incoherence in a proposition is logically a margin of logical incoherence. Logical coherence and moral coherence are directly proportional to Theoretical Reliability and to the percent reliability in critical reason. Theoretical Reliability equals one minus the inverse of the sample—whether the inverse of N in universes of infinite subjects or options, or the inverse of the direct scores or frequencies in universes of limited options. Moral incoherence is directly proportional to logical incoherence, and both are directly proportional to the inverse of the sample and to the margin of error accepted by critical reason. A claim is said to be logically coherent within a margin of error insofar as it is a coherent explanation of reality while accepting a margin of error; the smaller the margin of error, the greater the coherence. The relationship between error and coherence is inversely proportional: the greater the error, the lesser the coherence. Coherence is relative to moral stringency, and moral stringency is relative to reliability. Moral and logical coherence are directly proportional to Reliability, within the margin of error accepted by scientific policy.

6.2.2. Even if, in critical reasoning, only a percentage X of error equal to zero is accepted, or a percentage X of reliability equal to one hundred percent reliable, the universal inference for the explanation or understanding of the universe will be provisional, insofar as the probability of sample representativeness error will always remain in force—the inversion of the sample—whether the inversion of N in universes of subjects or infinite options, or the inversion of the direct scores or frequencies in universes of limited options. In population studies, including those within universes of subjects or infinite options, the inversion of the sample will likewise be the margin of sample representativeness error, even though, contradictorily, it may appear otherwise. Apparently, if no sample is selected from a population, it may seem there is no reason for a sample representativeness error, but this is only an appearance, because the population itself is the sample. In a study of ice volume by regions of Antarctica and the Arctic, where the population equals the regions established for the object of study, as

the ice-covered regions tend to shrink with increasing climate change, the study will lose representativeness to be a theoretically reliable description of the normal behavior of the sample. As a population tends to shrink, it loses representativeness; thus the sample representativeness error increases. In population studies the population is the sample itself, and likewise, as the population grows, sample representativeness will increase and the representativeness error will decrease. The world's population at any moment in its history is a sample of the global population's behavior at that moment in history; in a population that has a history, any study of the population at one moment in its history is the sample of the population's behavior at that moment in history, whose sample representativeness error will equal the inversion of the complete population. The provisional nature of knowledge is directly proportional to the error that science is willing to accept; if anything, in population studies one should say that, when a complete population is taken as the sample itself, sample selection does not depend on scientific policy—the scientific policy must accept the population as it is, as the sample itself, with the inversion of the population being the sample representativeness error. The second margin of error is, logically, that derived from critical reasoning. Every provisional universal synthesis will be, provisionally, a moral truth proportional to the margin of error—the margin of empirical skepticism or rational doubt—the moral responsibility of science.

6.2.3. Statistical probability, or the statistics of probability—Impossible Probability—formulates at least two types of propositions: provisional universal maxims deduced from statistical inference over margins of error, and historical universal maxims deduced from the description of the universe in history. In any case, whether inferential or descriptive, they are always relative to the very inevitability of history. A provisional universal maxim is one which, despite being provisional, is universally accepted during the provisional time in which it is accepted; in any case, every provisional universal maxim is essentially a provisional maxim. A historical universal maxim is a maxim universally accepted concerning the historical truth of a particular moment in history; in any case, every historical universal maxim is essentially a historical maxim. A provisional and historical universal maxim is a maxim universally accepted, provisionally, about a particular moment in history. In any case, everything

provisional—as it relates to a future time—is likewise historical, albeit a history yet to be demonstrated.

6.3. Validity

6.3.1. Concept of Validity

6.3.1.1. As has been explained, Validity is the comparison of the Level of Bias in relation to an ideal critical rationale, the critical probability, which will be calculated based on percentage X—of error or reliability—depending on whether it is an error or reliability study. In Validity of Equality it will be the difference of critical probability minus the absolute value of the Level of Bias. In Validity of Positive Bias, the difference of Level of Bias minus critical probability. In Validity of Negative Bias, the difference of the inversion of N minus empirical probability, and that result minus critical probability.

6.3.1.2. The critical probability, in the critical reasoning of Validity, will have different functions. In Validity of Equality, the critical probability will be the margin of error; in Validity of Bias, Positive or Negative, it will be the margin of reliability. Normally, the critical probability will be percentage X—of error or reliability—divided by one hundred, times a maximum or ideal trend.

6.3.1.3. The ideal depends on the object of study; to each reality there corresponds an idea and an ideal. Mathematics is the idealization of reality for the realization of the ideal. For an ideal of error, the ideal is to accept the minimum possible error. For an ideal of reliability, the ideal is to accept the maximum possible reliability. Scientifically, the first political decision is the acceptance of the margins of error: the margin of error derived from the sample, and the margin of error derived from critical reasoning. Ideologically, the first political decision is to determine the ideological ideal of the ideological policy, so that scientific policy tends to realize the ideal of the ideological policy in reality itself. The ideals of ideological policy will likewise determine the margins of moral responsibility to be accepted by scientific policy. The margin of error—a synthesis of the margin of logical incoherence and the margin of moral incoherence—is a margin of error established on the ideological politics of scientific policy so that scientific policy accepts the ideological hypothesis of the ideological policy. Every moral truth accepted with a margin of error is the acceptance of a margin of logical and

moral coherence within that margin of error, within which perhaps, possibly, the ideological hypothesis is not entirely coherent either logically or morally.

6.3.2. Validity of Equality, in studies of equality of opportunities: critical probability minus the absolute value of the Level of Bias; only zero or positive will be accepted. The critical probability will equal percentage X of error, divided by one hundred, times the Maximum Theoretical Possible Bias, with the Maximum Theoretical Possible Bias equal to the difference of one minus the inversion of N. The way the critical probability is symbolized is with the symbol “p(xc)”; thus whenever the symbol “p(xc)” appears it will symbolize the critical probability.

Validity of Equality = p(xc) – / (p(xi) – 1/N) / = zero or positive is accepted in equality of opportunities

p(xc) = (1 – 1/N) · (X : 100)
X = percentage of error

6.3.3. Validity of Positive Bias, in order to study whether the positive bias of the ideal subject or option is rationally sufficient: Level of Bias minus critical probability; only zero or positive is accepted. The critical probability will equal the product of percentage X of reliability, divided by one hundred, times the Maximum Theoretical Possible Bias, with the Maximum Theoretical Possible Bias equal to the difference of one minus the inversion of N.

Validity of Positive Bias = (p(xi) – 1/N) – p(xc) = zero or positive is accepted in a study of positive bias

p(xc) = (1 – 1/N) · (X : 100)

X = percentage of reliability

6.3.4. Validity of Negative Bias, only when the empirical probability is lower than the inversion of N: the difference of the inversion of N minus the empirical probability, and that result minus the critical probability; only zero or positive is accepted. The critical probability will equal the product of percentage X of reliability, divided by one hundred, times the inversion of N.

Validity of Negative Bias = (1/N – p(xi)) – p(xc) = zero or positive is accepted in a study of negative bias

p(xc) = 1/N · (X : 100)

X = percentage of reliability

6.4. Average Validity of Equality

6.4.1. Concept of Average Validity of Equality

6.4.1.1. Of all the Validity models, there will be one in particular for which computing the average can yield significant information about the behavior of the sample.

6.4.1.2. Validity of Equality equals critical probability minus the absolute value of the Level of Bias; zero or positive is accepted. Critical probability equals the product of

percentage X of error, divided by one hundred, times the difference of one minus the inversion of N—the Maximum Theoretical Possible Bias—which, for a study of equality of opportunities, is interpreted as the maximum error, of which only the minimum necessary proportion is accepted.

6.4.1.3. Whereas in a normal study of positive bias one assesses whether that single ideal in which the direct score or frequency must be increased reaches the maximum, and then the others tend to the minimum,

6.4.1.4. In a study of equality of opportunities the ideal is that all subjects or options that make up the sample, equally, come to be identical to the ineversion of N, within the margin of error that one is willing to accept—the critical reasoning of scientific policy.

6.4.1.5. Since in a study of positive bias only the Validity of Positive Bias of that single ideal defined as ideal by scientific policy will be zero or positive, it makes no sense to estimate the Average Validity of Positive Bias, given that under conditions of maximum possible dispersion—because there is a single ideal in the entire sample—it is only deemed necessary that a single subject or option reach zero or positive in Validity of Positive Bias; necessarily, any average value would make no sense.

6.4.1.6. In a study of negative bias, if an increasing number of subjects or options are defined as non-ideal, or the ideal is to increase the negative bias of that ideal, insofar as the critical reasoning will be to determine, among all negative biases, which is the greatest of all; as percentage X of reliability, divided by one hundred, times the inversion of N, is a sufficiently demanding percentage of reliability—accepting only the smallest possible margin of error, i.e., the smallest possible difference between one hundred and percentage X of reliability—logically, only those levels of negative bias will be accepted which, in Validity of Negative Bias, are zero or positive under a margin of reliability that accepts only the smallest possible error. Thus, of all the negative biases, only those that prove to be sufficiently rational will be zero or positive in Validity of Negative Bias; therefore, the average of all Validity of Negative Bias need not be positive, because, of all negative biases, only those that prove to be sufficiently rational will be accepted.

6.4.1.7. Since an average value of Validity of Positive Bias makes no sense, and the average of all Validity of Negative Bias need not be positive, then averages of Positive or Negative Bias Validity across the whole sample make no sense, because these are models whose object is to decide which one is truly sufficiently rational; of all the possibilities, only a minority—or only one—will be sufficiently rational. Therefore, the average of the sum of Validity of Positive or Negative Bias need not result in a zero or positive value.

6.4.1.8. However, if the object of study is equality of opportunities—an absolute value of the Level of Bias in every subject or option that is equal to or less than the critical rationale—the Average Validity of Equality does make sense: if, equally, every subject or option in N tends to the inversion of N, in any type of universe, every Validity of Equality tends to zero or positive. Therefore, an Average Validity of Equality that is zero or positive will demonstrate at least a sufficient tendency in the sample toward the inversion of N.

6.4.2. Average Validity of Equality

Average Validity of Equality = $\sum$ [p(xc) – / (p(xi) – 1/N) /] : N = zero or positive is accepted in equality of opportunities

p(xc) = (1 – 1/N) · (X : 100)

X = percentage of error

6.5. The error inherent to statistical inference

6.5.1. Every statistical inference will always carry two possible errors: the margin of error derived from the sample itself—the inversion of the sample (the inversion of N in universes of subjects or infinite options; the inversion of the direct scores or frequencies in universes of limited options)—and the percentage of error accepted by critical reasoning, whether percentage X of error in error studies, or one hundred minus

percentage X of reliability in reliability studies. In the critical appraisal of the trend, the ideal critical rationale must be the critical probability.

6.5.2. Even if critical reasoning accepted zero percent error in an error study, or one hundred percent reliability in a reliability study—even in the case where one accepts an ideal critical rationale of zero percent error or one hundred percent reliability—the provisional nature of the hypothesis will always, absolutely always, be directly proportional to the margin of sample representativeness error, which will entail accepting a moral risk—error—of taking as true a proposition that involves a possible error: taking as true a proposition that appears true but may actually be false. For the synthetic sciences, the only universal truth is the need to accept empirical propositions within a possible margin of logical or moral incoherence.

6.5.3. Every sample is a sample of time; every sample is a sample in a space; every sample is a *sample of space-time*. What is impossible in one particular space-time sample may even be inevitable in another different space-time sample. What is possible in one space-time sample need not be possible in any other space-time sample. Every sample of the infinite universe is only a possible sample within the infinite possible samples of that same infinite universe; and something may even have been possible in the sample's space-time but excluded by the sampling technique if one was required.

6.5.4. If a possible universe is truly infinite in all its dimensions, then any human knowledge of the universe can only be partial—never impartial—in every case ideological, in any case limited, and absolutely always merely one possible knowledge among possible knowledges. All possible knowledge can only be a provisional or historical maxim—relative, never absolute. In any case, only scientific work makes the universe absolutely partial for itself, because the universe in itself will always be universal; outside itself the universe is an electrochemical appearance—material or cerebral—in any case empirical, and in every case an object of critical reasoning concerning the margin of empirical skepticism or rational doubt—moral responsibility toward the truth—we are willing to accept in explaining what happens, perhaps only apparently, within the random causal indeterminacy of the insignificant—error. Every possible error, in rational or sufficient time, is logically an error. To accept an error is

logically an error dialectically necessary. The cause–effect identity is logically dialectical, or dialectically logical, within skepticism, doubt, or responsibility before possible causes that perhaps are not causes, and the very idea of cause itself may be a purely *physiological* appearance.

6.6. The Critical Sample Level

6.6.1. Concept of the Critical Sample Level

6.6.1.1. One of the principal qualities of Impossible Probability is equifinality: the same reality can be appraised by different models of critical reasoning. Validity of Equality or of Bias (Positive or Negative) is one model of critical reasoning among others explained in this work, but they are, in any case, models of individual critique.

6.6.1.2. The fact that in Validity of Equality, or Positive Bias, or Negative Bias, the Validity is zero or positive—hence accepted—only implies that an individual trend is accepted as truly individual; by itself it does not justify statistical inference, because, absent knowledge of the sample trend, all that is shown is a particular individual trend that might have been caused by the sample's magnitude error.

6.6.1.3. The inversion of the sample—whether N in universes of subjects or infinite options, or the sum of direct scores or frequencies in universes of limited options—measures, at the same time, theoretical error and theoretical dispersion.

6.6.1.4. By means of Sample Sufficiency (according to the type of universe)—critical appraisal of whether the inversion of the sample (N or direct scores/frequencies) is equal to or less than a critical rationale—or by means of the Critical Normality Requirement (in positive bias or certain equality studies) or the Normality Requirement (if the object is moderation and for any equality study), one studies whether to accept the representativeness error or theoretical bias. However, the fact that a sample is accepted as sufficient does not imply the absence of error, because every acceptance of an error is the acceptance of a possible error; accepting an error will always be an error, albeit, for the moment, only a possible one—in theory.

6.6.1.5. Everything—absolutely everything—possible, including error, in sufficient or infinite time, can be inevitable. Accepting an error is accepting an error—a margin of

empirical skepticism or rational doubt. Accepting an error entails moral responsibility—practical reason.

6.6.1.6. Moreover, each time one studies Validity—of Equality or Bias, Positive or Negative—one not only accepts the margin of sample representativeness error; one also accepts a margin of rational error via the critical probability through which the individual trend is accepted.

6.6.1.7. One way to determine, in inference, that acceptance of an individual trend is not a direct product of the sample representativeness error is, once the individual trend is accepted, to critique rationally the trend observed in the sample—i.e., to critique the trend at the sample level—in order to rule out a possible error in fact in the statistical decision, within the margin of factual error we are willing to accept. In fact, errors are always accepted; the question is whether that factual error falls within what is politically acceptable, so that, in the syllogism of trend—given an individual and a sample trend that are sufficiently rational—one can infer the **behavior of the universe**.

6.6.1.8. If the sample trend is essentially the Mean or Typical Deviation, one way to ensure that the individual trend is sufficient to be universal is to verify that the sample trend proves sufficient against the critical rationale.

6.6.1.9. Since the sample trend—Mean or Typical Deviation—is inversely proportional to the magnitude of the sample, in the absence of a critical appraisal of the sample trend it might happen that we accept a phenomenon as due to equality of opportunities (because a Validity of Equality is zero or positive) when the tendency of the Level of Bias toward zero has been produced by the magnitude of the sample. A sufficiently large sample can produce a tendency toward zero of the sample dispersion, causing us to accept an individual tendency to equality of opportunities when that tendency may have been caused directly by the sample's magnitude, and not necessarily because the subject or option is truly sufficiently rational.

6.6.1.10. In bias studies exactly the same may occur: one may accept that an individual trend—under Validity of Positive or Negative Bias—is sufficiently rational to be accepted, when it could be the product of an insufficiently demanding margin of error in the sample's magnitude, failing to reduce the representativeness error enough,

thereby allowing an increase in bias as a consequence of the sample's magnitude, which may cause individual biases to increase due to the representativeness error itself, and not because the individual biases are truly rational.

6.6.1.11. Since situations can arise in which an individual trend is accepted when that trend has been produced by the sample's magnitude—and not because the subject or option truly follows a rational behavior model—one way to validate at the sample level that an individual trend is truly individual and not a product of the sample's magnitude is through the critical appraisal of the sample's own trend, i.e., the critical appraisal of the Mean or Typical Deviation.

6.6.1.12. If, in equality of opportunities, the ideal is that the Mean or Typical Deviation tends to zero, then the ideal is that, at the Critical Sample Level, the Mean or Typical Deviation be equal to or less than an ideal critical rationale.

6.6.1.13. If, in a bias study—positive or negative—the ideal is that the Mean or Typical Deviation increases, then the ideal is that, at the Critical Sample Level, the Mean or Typical Deviation be equal to or greater than a critical rationale.

6.6.1.14. Since the critical probability is explained in Section Eleven, in this section it will simply be stated directly, although Section Ten already explains how to determine the critical probability for Mean Deviation.

6.6.1.15. In essence, for the Critical Sample Level of Equality: critical probability minus Mean or Typical Deviation—zero or positive is accepted. The critical probability equals percentage X of error, divided by one hundred, times the Maximum Theoretical Possible Mean Deviation (if the Critical Sample Level is set on Mean Deviation); or, if set on Typical Deviation, the critical probability equals percentage X of error, divided by one hundred, times the Maximum Theoretical Possible Typical Deviation.

6.6.1.16. Critical Sample Level of Bias: Mean or Typical Deviation minus critical probability—zero or positive is accepted. The critical probability equals percentage X of reliability, divided by one hundred, times the Maximum Theoretical Possible Mean Deviation, or the Maximum Theoretical Possible Typical Deviation, depending on whether it is computed on Mean or Typical Deviation.

6.6.1.17. The inversion of N is considered an operational unit in itself; that is, the reciprocal of N will always appear without being enclosed in parentheses or brackets so as not to overload equations, but whenever the inversion of N, 1/N, appears as is—in sums, differences, products, quotients, or, for example, squares—it must be interpreted as, for example, "the square of (one divided by N)," or "the sum/difference/product, wherever it appears, of the result of one divided by N." [In the original I never put the 1/N in brackets, in this English version I will do it, to be formal, so in this version the square of inversion of N will appear in brackets $(1/N)^2$]

6.6.2. Critical Sample Level of Equality: critical probability minus Mean or Typical Deviation; zero or positive is accepted.

Critical Sample Level of Equality = p(xc) – DMoS = zero or positive is accepted in a study of equality of opportunities

6.6.2.1. Critical Sample Level of Equality via Mean Deviation (DM)

Critical Sample Level of Equality = p(xc) – DM = zero or positive is accepted in a study of equality of opportunities

$$p(xc) = \{ [(1 – 1/N) \cdot 2] : N \} \cdot (X : 100)$$

X = percentage of error

6.6.2.2. Critical Sample Level of Equality via Typical Deviation

Critical Sample Level of Equality = p(xc) – S = zero or positive is accepted in a study of equality of opportunities

$$p(xc) = \sqrt{\{ \{ [1 - 1/N]^2 + [(1/N)^2 \cdot (N - 1)] \} : N \} \cdot (X : 100)}$$

X = percentage of error

6.6.3. Critical Sample Level of Bias: Mean or Typical Deviation minus critical probability; zero or positive is accepted.

Critical Sample Level of Bias = DMoS – p(xc) = zero or positive is accepted in a bias study

6.6.3.1. Critical Sample Level of Bias via Mean Deviation

Critical Sample Level of Bias = DM – p(xc) = zero or positive is accepted in a bias study

$$p(xc) = \{ [(1 - 1/N) \cdot 2] : N \} \cdot (X : 100)$$

X = percentage of reliability

6.6.3.2. Critical Sample Level of Bias via Typical Deviation

Critical Sample Level of Bias = S – p(xc) = zero or positive is accepted in a bias study

$$p(xc) = \sqrt{\{ \{ [1 - 1/N]^2 + [(1/N)^2 \cdot (N - 1)] \} : N \} \cdot (X : 100)}$$

X = percentage of reliability

6.6.4. Applications of the Sample Level in statistical inference

6.6.4.1. Statistical inference of equality

6.6.4.1.1. Although in *Introduction to Impossible Probability*—because of equifinality—the trend toward equality of opportunity can be critiqued with different models (as in the critical appraisal of bias), one of them, the first to appear in Impossible Probability, is the Validity of Equality. Since the Validity of Equality only determines whether, at the individual level for each subject or option, there is a sufficient trend toward equality of opportunity—a trend that can be influenced by the random error of the sample's magnitude—once, in the Validity of Equality, a majority of subjects or options show zero or positive, or zero or positive is observed in the Average Validity of Equality, one way to determine that this trend has not actually been caused by a factual error (within the margin of error we are willing to accept) is through the Critical Sample Level of Equality. Thus, insofar as the Mean or Typical Deviation is equal to or less than a critical probability that tends sufficiently toward zero (critical probability = percentage X of error, tending toward zero, divided by one hundred, times the Maximum Theoretical Possible Deviation—Mean or Typical), one will accept that the individual trend observed in the sample is critically demonstrated at the sample level.

(p(xc)≈0)−DMoS=zero or positive → equality at the sample level is accepted

6.6.4.1.2. The closer the critical probability is to zero in the Critical Sample Level of Equality, the more morally demanding the test will be (the less error is accepted); and the farther the critical probability is from zero, the more error is accepted and the lower the reliability and moral stringency of the hypothesis test to accept, at the sample level, a sufficient trend toward equality of opportunity.

6.6.4.1.3. The critical probability will be closer to zero as the product of percentage X of error we are willing to accept divided by 100, and multiplied by the Maximum Theoretical Possible Deviation—Mean or Typical—approaches zero, thus being more morally demanding and reliable. The critical probability will be less close to zero as that product is less close to zero error, making the test less reliable and morally less stringent.

6.6.4.2. Statistical inference of bias

6.6.4.2.1. For statistical inference of whether a positive bias in an ideal subject or option—or a negative bias in a non-ideal subject or option, or where the ideal is to tend to the minimum—is sufficient (both at the individual level and at the sample level), logically the only way at the sample level to validate that the observed individual trend of positive or negative bias is sufficient to be rational is to demonstrate, at the sample level, that the trend toward increasing bias in the sample—resulting from the increase of that particular positive or negative bias, or of those positive or negative biases in general—produces a sufficient increase in empirical sample dispersion such that the Mean or Typical Deviation is equal to or greater than a critical probability close to the Maximum Theoretical Possible Deviation—Mean or Typical.

DM−(p(xc) ≈{ [(1−1/N)·2] :N }) = zero or positive → bias accepted at the sample level\

$S-(p(xc) \approx \sqrt{\{ [(1-1/N)^2 + ((1/N)^2 \cdot (N-1))] : N \}) =$

zero or positive → bias accepted at the sample

6.6.4.2.2. The closer the percentage X of reliability divided by 100, and then multiplied by Maximum Theoretical Possible Deviation—Mean or Typical—is to one hundred percent (hence the critical probability is closer to the Maximum Theoretical Possible Deviation—Mean or Typical), the greater the reliability and moral stringency and the smaller the rational error accepted in the statistical decision—therefore, more reliable. The less that percentage of reliability approaches one hundred percent, the more rational error is accepted and the less reliable (and morally less stringent) the statistical decision.

6.6.4.2.3. Logically, in the Critical Sample Level of Bias, the ideal would be to accept only those Mean or Typical Deviations that are equal to or greater than a critical probability based on the highest possible percentage X of reliability, thus accepting the smallest factual and rational error.

6.6.4.3. Modalities of empirical error and reliability

6.6.4.3.1. As explained in Section Sixteen, there will be three modalities of empirical error or reliability. The first two apply to normal studies—when dispersion ranges between zero and maximum. The ratio of sample dispersion to maximum sample dispersion will be interpreted as the First Modality of Empirical Error in normal equality studies, and as the Second Modality of Empirical Reliability in those studies where, out of N, there is only a single ideal. Then, one minus that ratio will be the First Modality of Empirical Reliability in equality studies, or the Second Modality of Empirical Error if there is only one ideal in N.

6.6.4.3.2. The Third Modality of Empirical Error, in omega (if the number of ideals in N is between two and N minus one), is: divide, by the ideal sample dispersion, the absolute

value of the differential (sample dispersion minus ideal sample dispersion). Then, one minus that quotient equals the Third Modality of Empirical Reliability.

6.7. Chance

6.7.1. The natural law is the law of chance.

6.7.2. The law of chance is the law of probabilities.

6.7.3. The law of probabilities is that, as the sample increases, everything tends to be identical to the inversion of N. The origin of error is the contradiction between infinite reality and human limitation. In limited universes, existence is inevitable; in universes of subjects or infinite options, even the impossible would be theoretically possible; in empirical reality—in practice—only what has logical coherence is possible.

6.7.4. Chance is natural goodness: free will under equality of opportunity.

6.7.5. The natural goodness of chance is the transformation of the positive into the negative and the negative into the positive—negation of the negation; normally, all positive bias compensates all negative bias.

6.7.6. In chance, negative bias and positive bias are identical. All positive bias normally entails one or several negative biases whose sum equals the positive bias. All negative bias normally entails one or several positive biases whose sum equals the negative bias. The magnitude of normal positive bias will be absolutely identical to the magnitude of normal negative bias, within the margin of error we are willing to accept: the inversion of the sample.

6.7.7. Normally: every occurrence of negative bias implies occurrences of positive bias, and vice versa; therefore, all negative biases are positive (because everything negative implies everything positive) and all positive biases are negative (because everything positive implies everything negative). The identity of the positive and the negative is a product of the *immanent* natural goodness of chance, within the margin of error we are willing to accept—the inversion of the sample (the inversion of N in universes of subjects or infinite options; the inversion of direct scores or frequencies in universes of limited options). The larger the sample, the smaller the normal tendency in the magnitude of negative bias; thus, with a larger sample, the smaller the normal tendency

in the magnitude of positive bias—normally, because all positive bias equals all negative bias. The smaller the sample, the greater the normal tendency to positive bias; thus, with a smaller sample, the greater the normal tendency to negative bias—normally, because all negative bias equals all positive bias. Everything negative and everything positive will always be identical within the margin of moral responsibility toward the sample—that part of reality in itself that we can only know partially. The utopia of impartial, neutral knowledge is impossible for science, which only knows reality partially, ideologically; impartial and **neutral knowledge of reality will only be possible when modern science is surpassed** [GAI] in the possible permanent revolution of history toward a new mathematical redefinition [non human pure operations] in which absolutely reliable knowledge is possible. As long as error is necessary, science can only be ideology—a semblance of reliability that knows nothing absolutely because it understands everything provisionally. The current mathematical definition, being incomplete for demonstrating its consistency, implies a mathematics that is partially complete, partially coherent, relatively objective, possibly inconsistent, possibly incoherent. Until the mathematical definition changes [with GAI], science will be dependent on an ideological mathematics—mathematical ideology.

6.7.8. Chance is freedom itself, equal for all possible occurrences, under conditions of equal opportunity.

6.7.9. In chance, the positive has the same freedom of occurrence as the negative, and vice versa.

6.7.10. In chance, positive and negative bias are identical, and at the same time chance and bias are identical.

6.7.11. The central tendency or dispersion of bias or chance is called a distribution. A distribution can be by bias, or a distribution can be by chance; in any case, all bias is a product of chance and all chance is a product of bias—the order of the factors does not alter the product. Mean Deviation is at the same time Total Bias times the inversion of N, or the inversion of N times Total Bias; in short: bias by chance or chance by bias.

6.7.12. Every scientific idea tends to be identical to reality; this is the origin of the isomorphism of the positive fact, within the margin of moral responsibility ***before historical transcendence.***

6.7.13. The identity of the distribution is an identity within the margin or percentage of skepticism or doubt. A distribution will be identified as a distribution by chance or by bias according to the margin of causal indeterminacy or logical uncertainty.

6.7.14. Chance, bias, and error are three logical moments of the same dialectical process.

6.7.15. Opposites are identical because they are opposed: chance and bias (or bias and chance), chance and error (or error and chance), bias and error (or error and bias)—each in relation to all, and all in relation to each—are identical within the margin of skepticism or uncertainty.

6.7.16. Impossibility, possibility, and inevitability are three logical moments of the same infinite dialectical process; nothing is what it seems outside itself.

6.7.17. In the truly infinite universe in all its dimensions, the possible and the impossible, the possible and the inevitable, the inevitable and the impossible, are identical—in *the tendency of every being toward nothing, zero,* and the tendency of everything to be determined by nothing: absolute causal indeterminacy, uncertainty in itself.

6.7.18. Freedom and equality are identical because they are opposites, within the margin of error, bias, or chance that critical reason is willing to accept for the practical realization of ideals.

6.7.19. Statistical probability is the study of what happens in *chance* and in *history*.